Large N QFT

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概要・動機

概要

Large N QFTとは・・・場の数N→∞の理論

AdS/CFTに関係

$\phi^i \quad \phi^{ij} \quad \phi^{ijk}$

$i, j\ k = 1,2, \ldots N$

$O(N) \quad O(N)^2 \quad O(N)^3 \quad U(N)$ …など

動機

・大学院でAdS/CFTをやりたい

・場の理論の知識がない

⇒ Large Nから勉強することに

アウトライン

1. 実スカラー場の理論　$\phi^4 model,\ \phi^3 model$（1枚）

2. Large Nアプローチ

   2.1　$Vector\ model：\ O(N)\ \ \phi^i$　　（3枚）

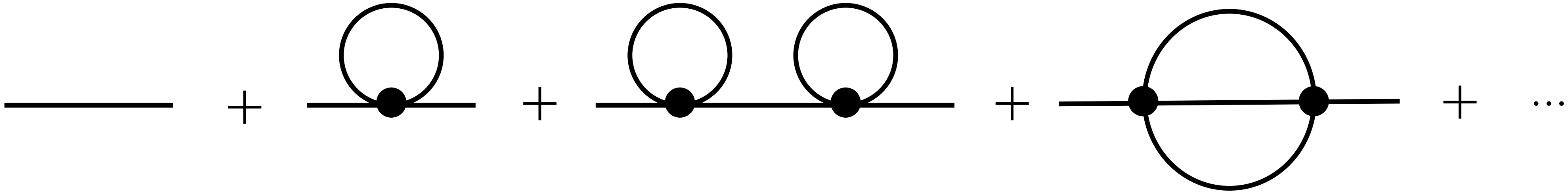
   2.2　$Matrix\ model : U(N)\ \ \Phi^i_j$　　（3枚）

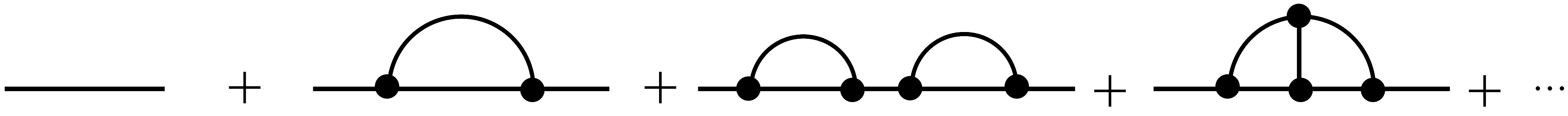
   2.2　$Tensor\ model : O(N)^3\ \ \phi^{ijk}$　　（3枚）

3. まとめ、今後の展望　　（2枚）

1 実スカラー場の理論, $\phi^4 model, \phi^3 model$

○相互作用が小さいとき→摂動論





○摂動のn次

→ $Feynman\ diagram$の数 $\sim n!$

摂動が収束しない ⇒ 't Hooft Large N 理論

$\phi^i \quad \Phi^i_j \quad \phi^{ijk}$

2.1 Vector model ϕ^i $(O(N))$

0+0次元

$$\mathcal{L} = -\frac{1}{2}\phi^i\phi^i - \frac{g}{4!}\phi^i\phi^i\ \phi^j\phi^j \qquad (i, j = 1,2,\ldots N)$$

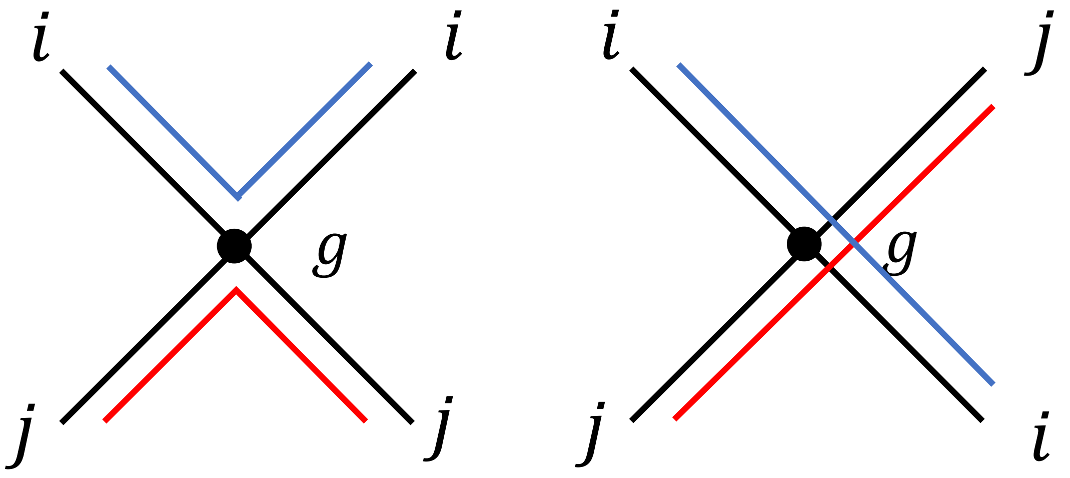
ϕ^iの添字iをカラーと呼ぶとする。

プロパゲーター

$$\langle\phi^i\phi^j\rangle = \delta^{ij}$$

カラーの流れが発生

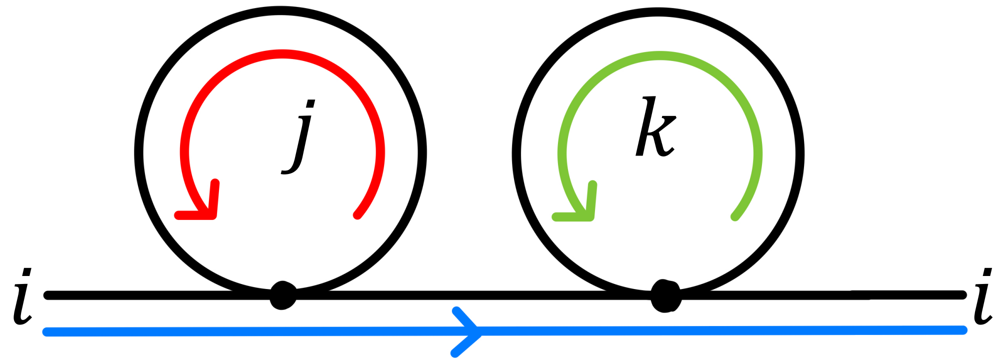
4点相互作用



2.1 $Vector\ model\quad \phi^i\quad (O(N))$

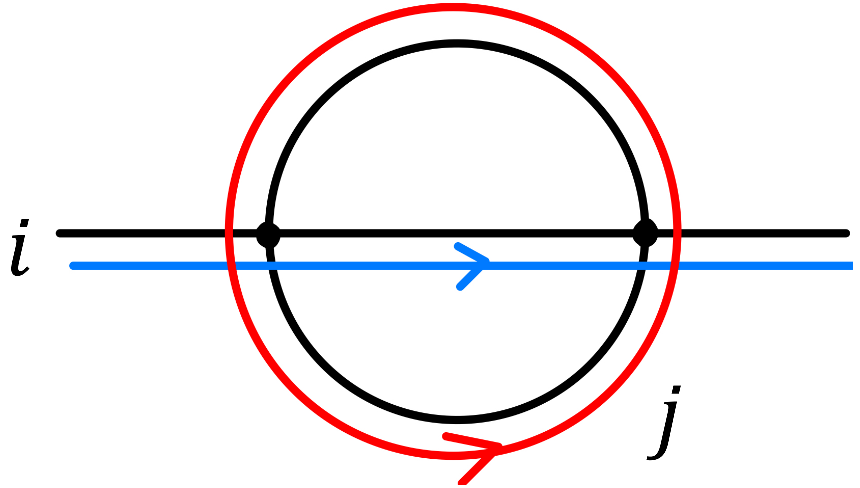
$$\underline{N \to \infty\ ,\quad \lambda \equiv gN\ \ fixed} \qquad (t'Hooft\ \ limit)$$

snail diagram



$$\sim g^2 N^2 = \lambda^2$$

melon diagram



$$\sim g^2 N = \frac{\lambda^2}{N}$$

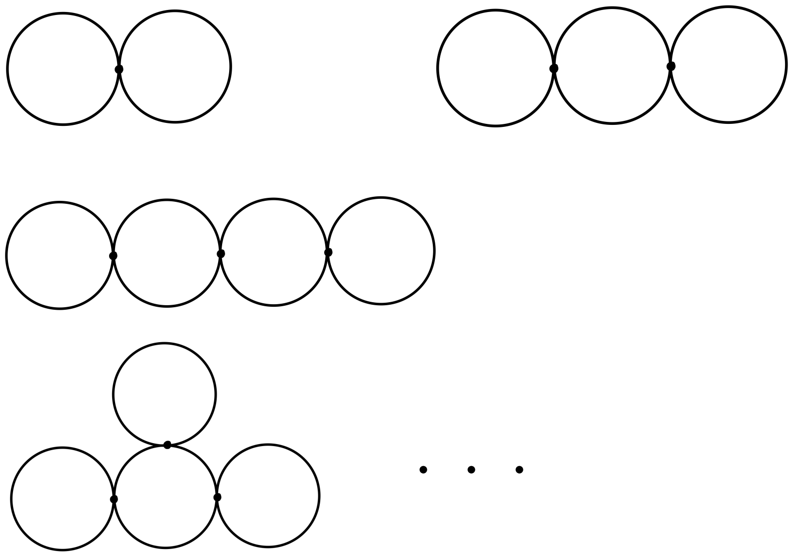
$$\to 0\ \ (N \to \infty)$$

2.1 $Vector\ model\ \ \phi^i\ \ (O(N))$

真空ダイアグラムの場合

○

$bubble\ diagram \sim O(\lambda^{\#}N)$



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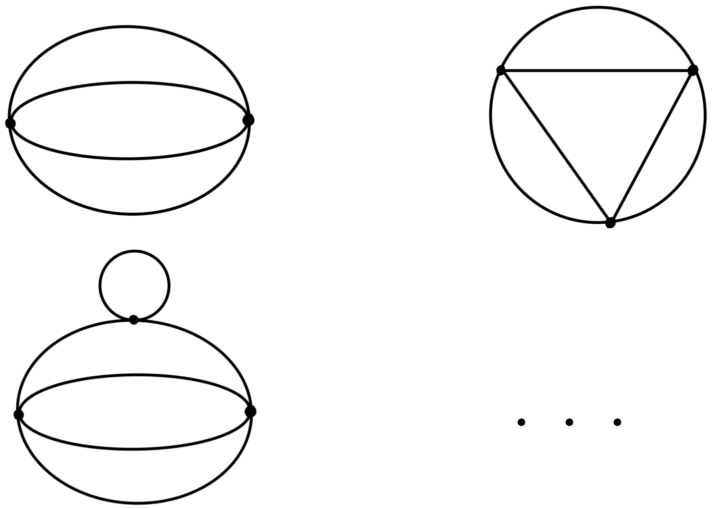
それ以外は高々$\sim O(\lambda^{\#})$



非摂動で解けるモデル　$Z(\vec{J}=0,\lambda)=\lambda$の関数

2.2 *Matrix model* Φ^i_j $(U(N))$

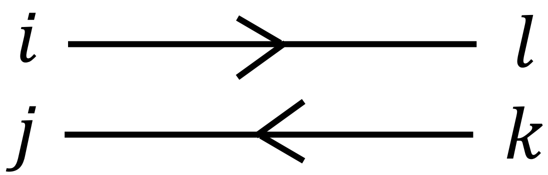
0+0次元

$$\mathcal{L} = -\frac{1}{2}tr(\Phi^2) - \frac{g}{3!}tr(\Phi^3)$$

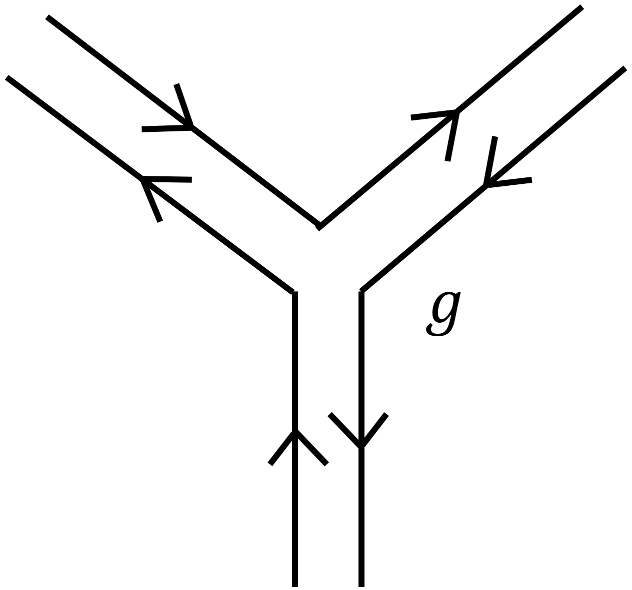
$(\Phi : N\times N\ matrix)$

プロパゲーター

$$\langle \Phi^i_j \Phi^k_l \rangle = \delta^i_l \delta^k_j$$

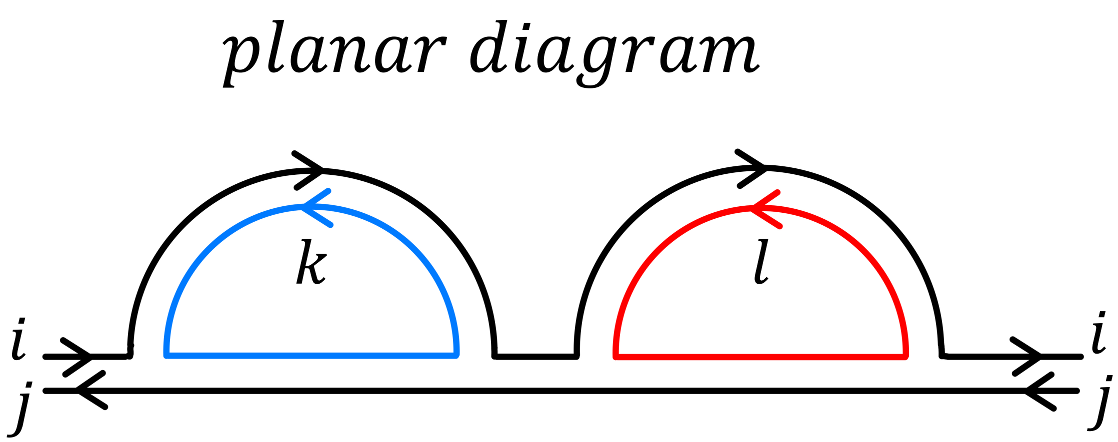


3点相互作用

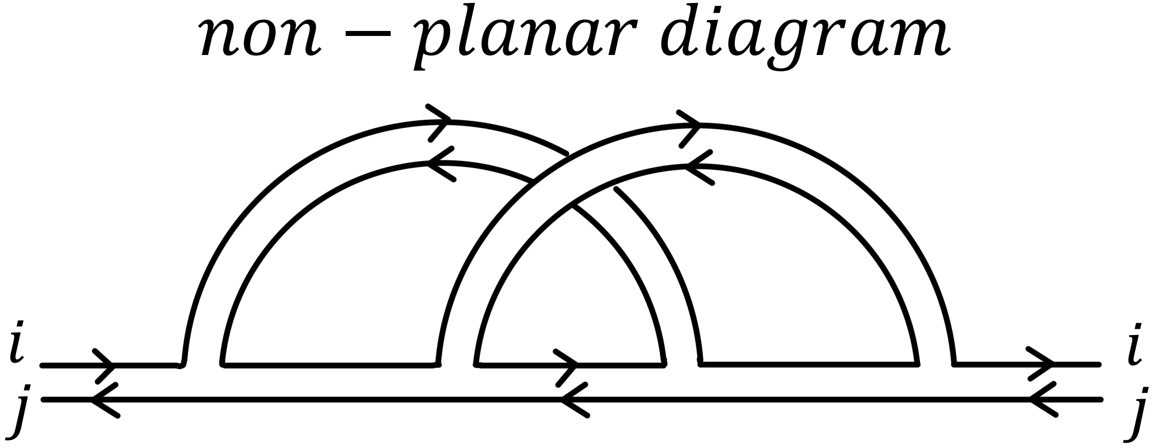


2.2 $Matrix\ model\ \ \Phi^i_j\ \ (U(N))$

$$N \to \infty\ ,\ \ \lambda \equiv g^2 N\ \ fixed \qquad (t'Hooft\ \ limit)$$



$$\sim g^4 N^2 = \lambda^2$$

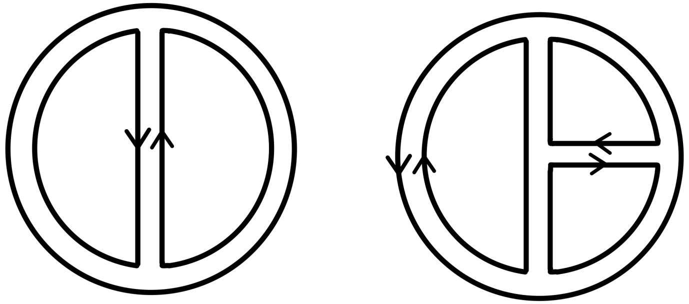


$$\sim g^4 = \lambda^2 N^{-2}$$

$$\to 0\ \ (N \to \infty)$$

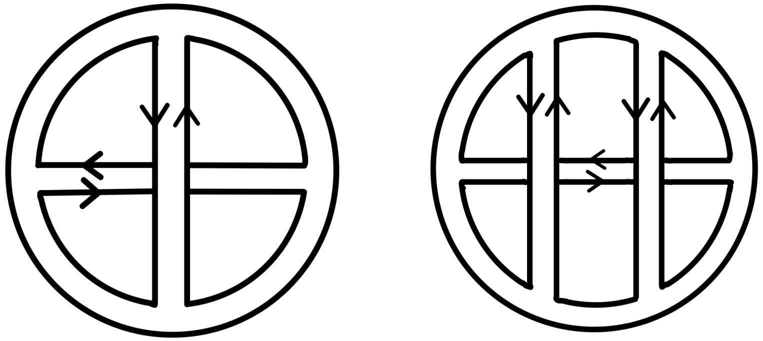
2.2 *Matrix model* Φ^i_j $(U(N))$

planar diagram



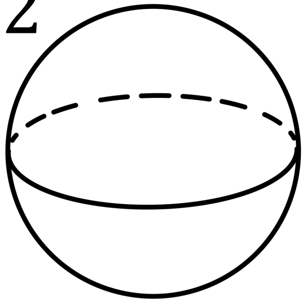
$\sim \lambda N^2$ $\sim \lambda^2 N^2$ . . .

non − planar diagram

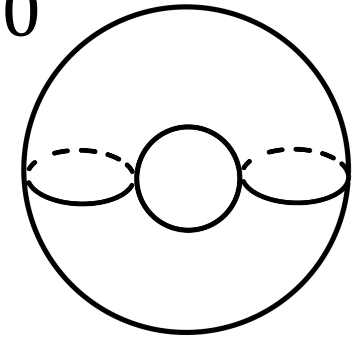


$\sim \lambda^2 N^0$ $\sim \lambda^3 N^0$ . . .

$\chi = 2$



$\chi = 0$



$\chi = -2$

. . .

(確率振幅)$\sim \lambda^{\#} N^{\chi}$

$(\chi = 2, 0, -2, \ldots)$

*planar diagram*しか効かない

2.3 *Tensor model* ϕ^{ijk} $(O(N)^3)$

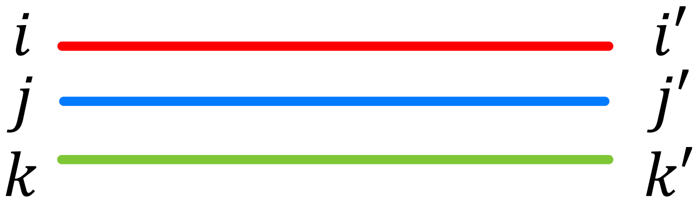
0+0次元

$$\mathcal{L} = -\frac{1}{2}\phi^{ijk}\phi^{ijk} - \frac{g}{4!}\phi^{i_1 j_1 k_1}\phi^{i_1 j_2 k_2}\,\phi^{i_2 j_1 k_2}\phi^{i_2 j_2 k_1}$$

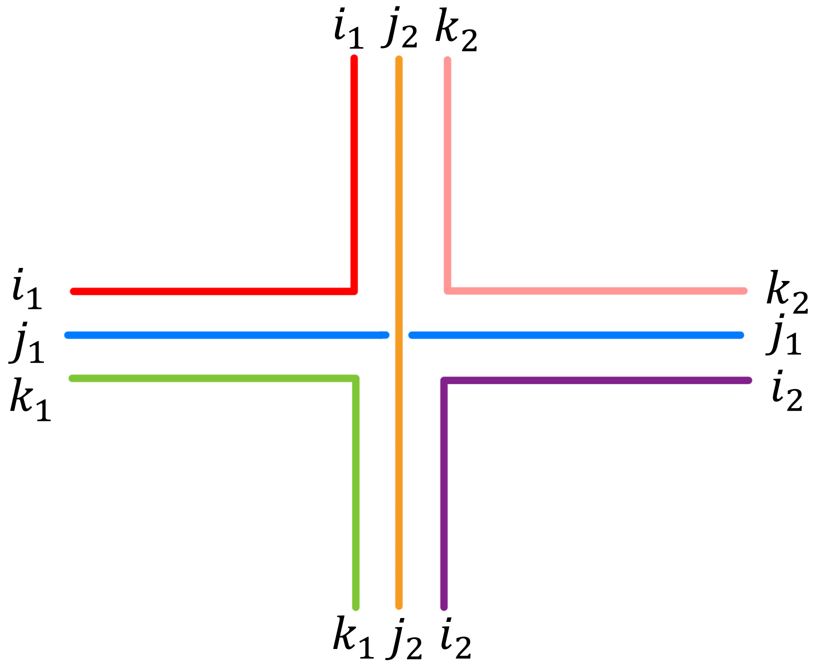
$(i, j, k, i_1 \ldots = 1,2, \ldots N)$

プロパゲーター

$$\langle \phi^{ijk}\phi^{i'j'k'} \rangle = \delta^{ii'}\delta^{jj'}\delta^{kk'}$$



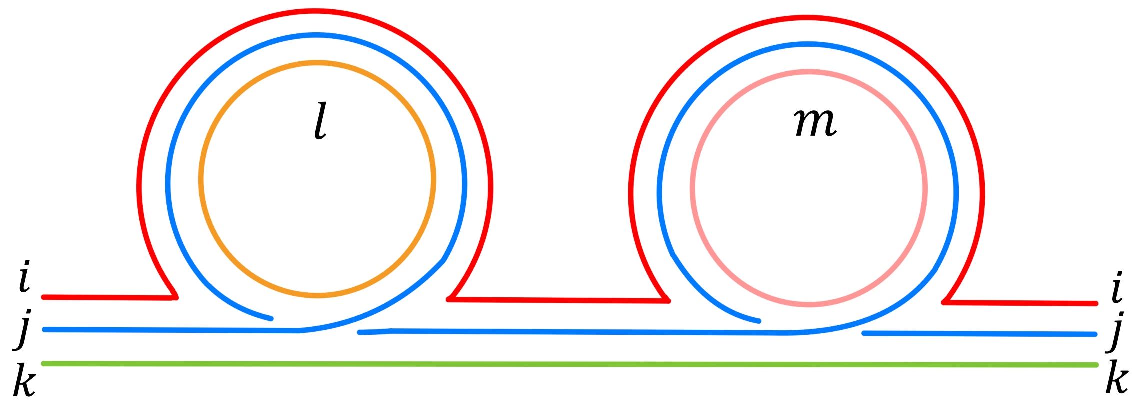
4点相互作用



2.3 $Tensor\ model\ \ \phi^{ijk}\ \ (O(N)^3)$

$$N \to \infty\ ,\ \ \lambda^2 \equiv g^2 N^3\ \ fixed \quad (t'Hooft\ \ limit)$$

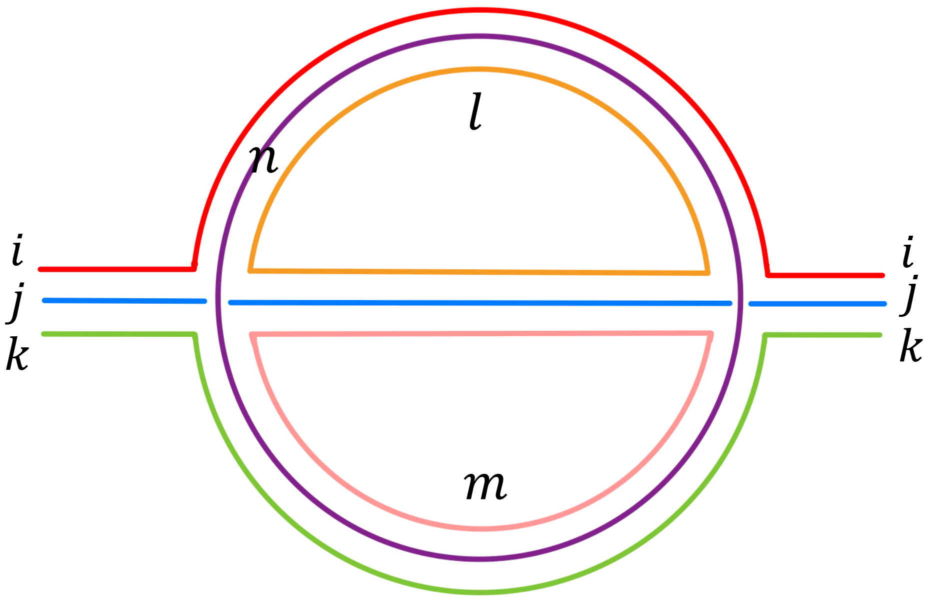
snail diagram



$$\sim g^2 N^2 = \frac{\lambda^2}{N}$$

$$\to 0\ \ (N \to \infty)$$

melon diagram



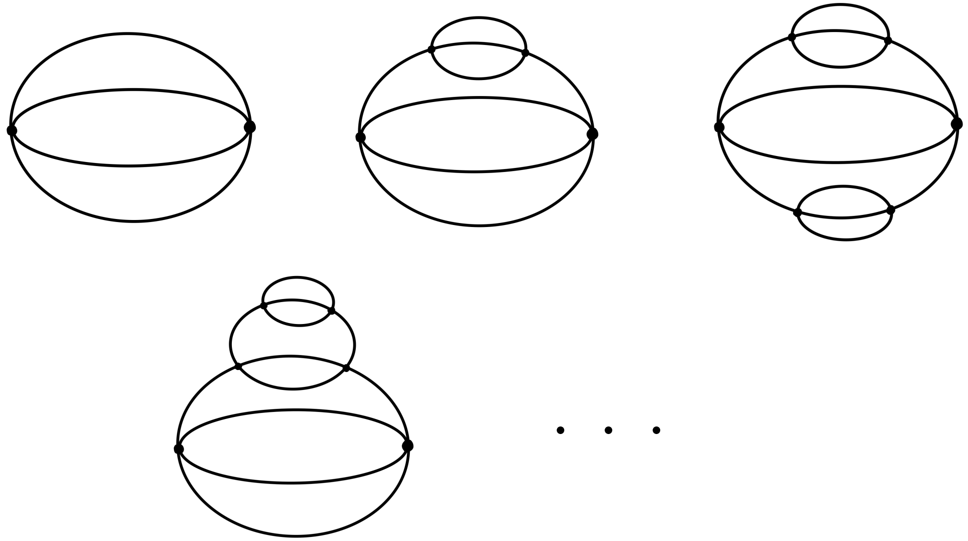
$$\sim g^2 N^3 = \lambda^2$$

2.3 *Tensor model* ϕ^{ijk} $(O(N)^3)$

真空ダイアグラムの場合

○

melon diagram$\sim O(\lambda^{\#} N^3)$



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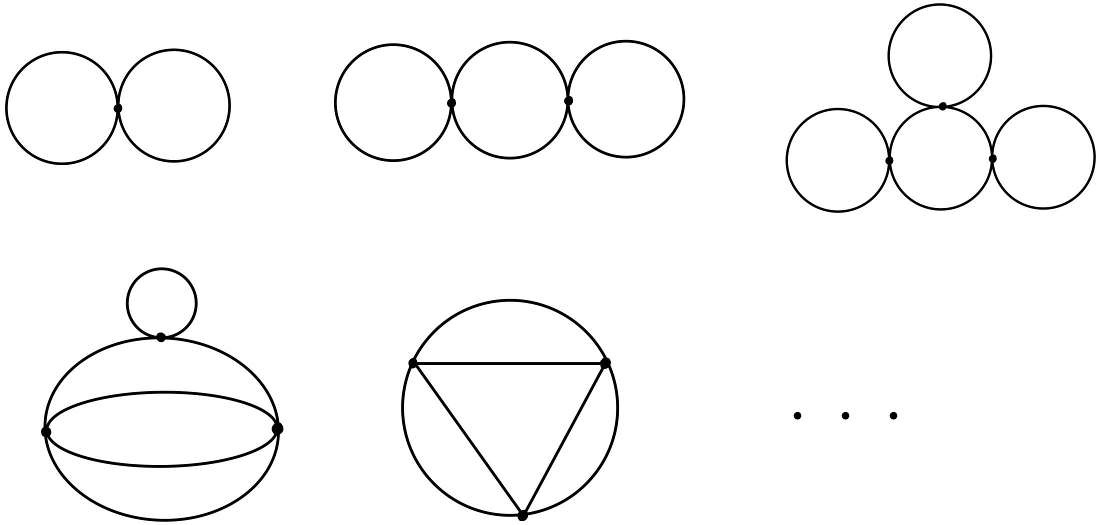
それ以外は高々$\sim O(\lambda^{\#} N^{5/2})$



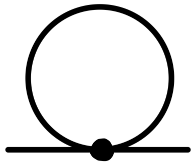
非摂動で解けるモデル　$Z(\vec{J}=0,\lambda)=\lambda$の関数

まとめ

○ 場の種類を増やすことでカラーの流れができた。

○ 場の数$N \to \infty$の極限では、一部の$Feynman\ diagram$しか効かない。

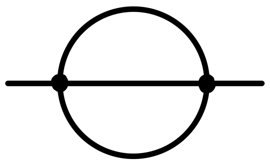
$Vector\ model \to snail\ diagram$



$Matrix\ model \to planar\ diagram$

○ トポロジーとの関係が現れた

$Tensor\ model \to melon\ diagram$



○ 摂動の収束が良くなり、非摂動で解ける場合もある。

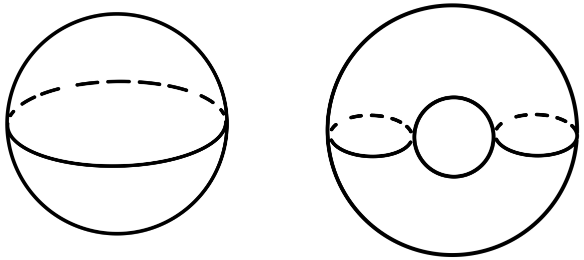
3. 今後の展望

卒研で勉強(中)← | →今後勉強 / 研究したい

Matrix model ⇒ Large N QCD

$SU(N)$ゲージ理論 ⇔ 弦理論 (ホログラフィー)

$$\mathcal{L}_{YM} = -\frac{1}{4} tr\left(F_{\mu\nu}F^{\mu\nu}\right)$$



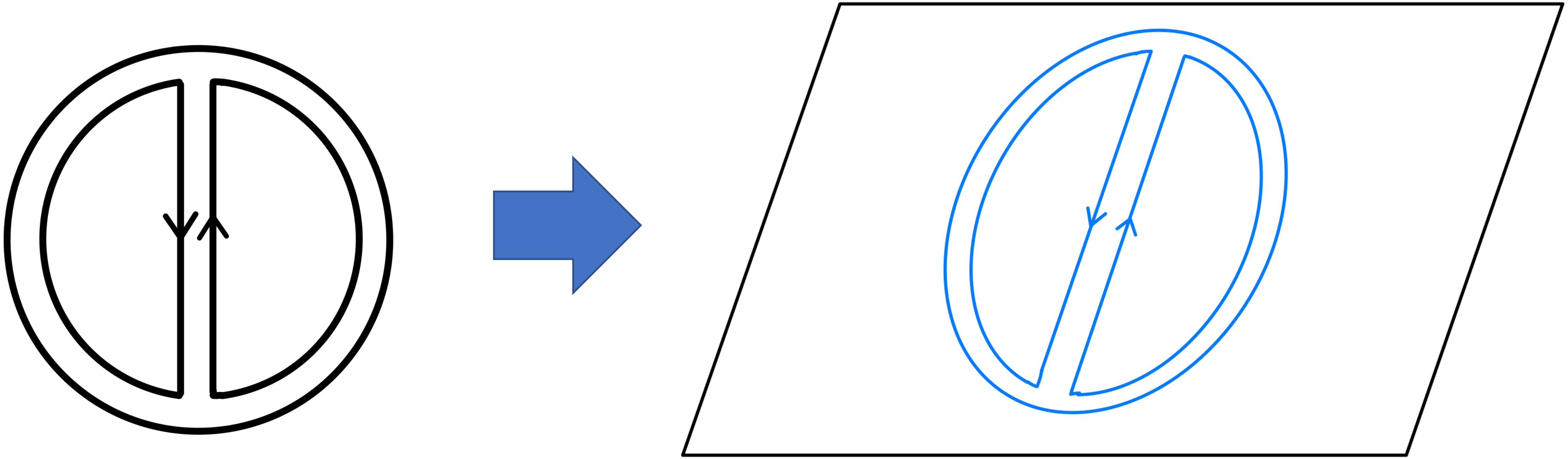
・・・

Tensor model (*melonic*) ⇒ Sachdev-Ye-Kitaev *SYK model* ⇔ AdS_2 、ブラックホール

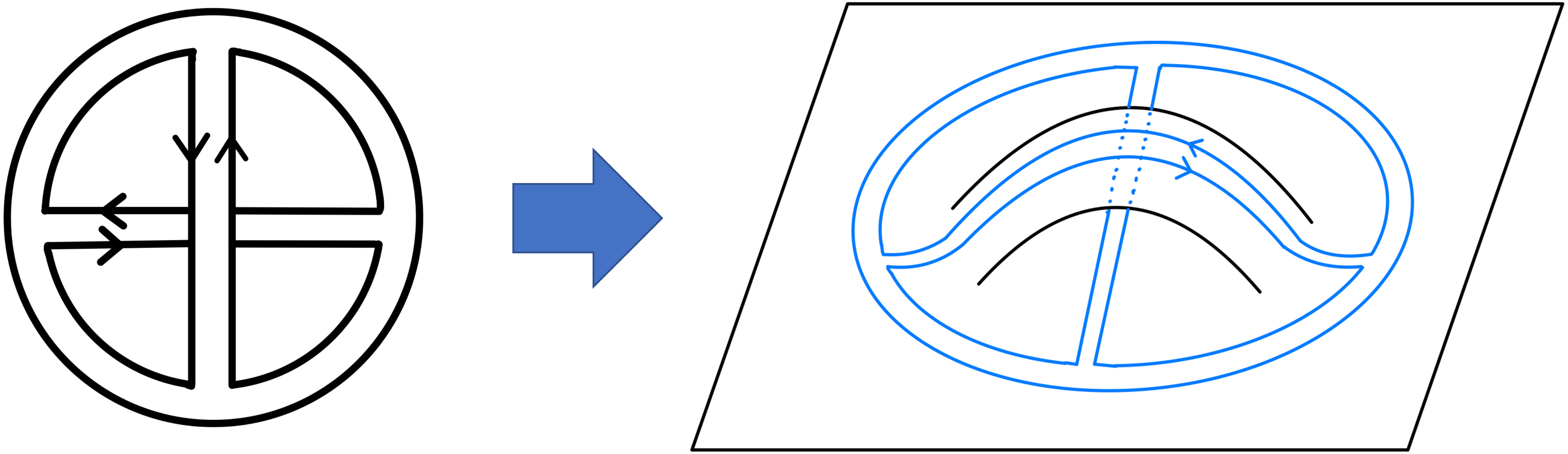
↑ Large N ,melonic, solvable

↑ 2次元重力

4. 補遺



4. 補遺



4. 補遺

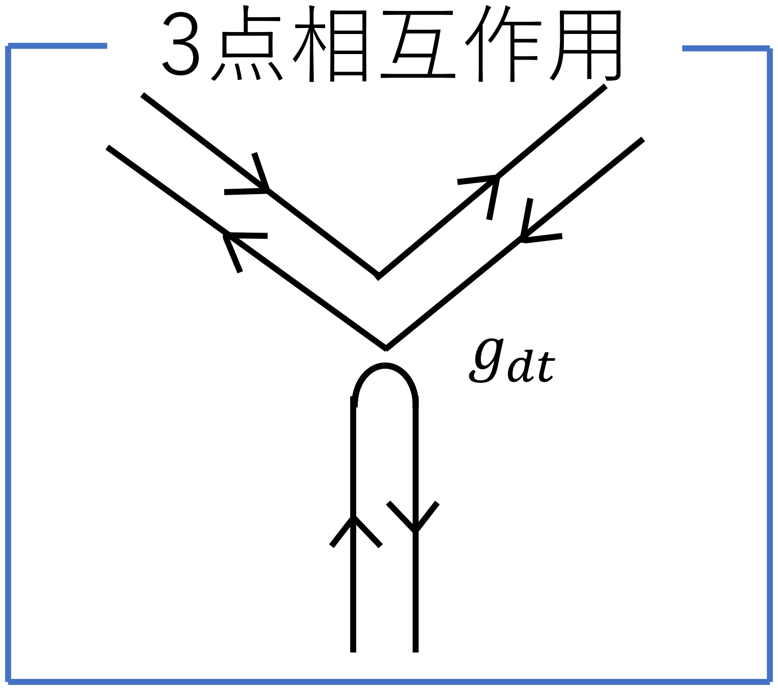
2.1 $Vector\ model\ (d = 0 + 0)$

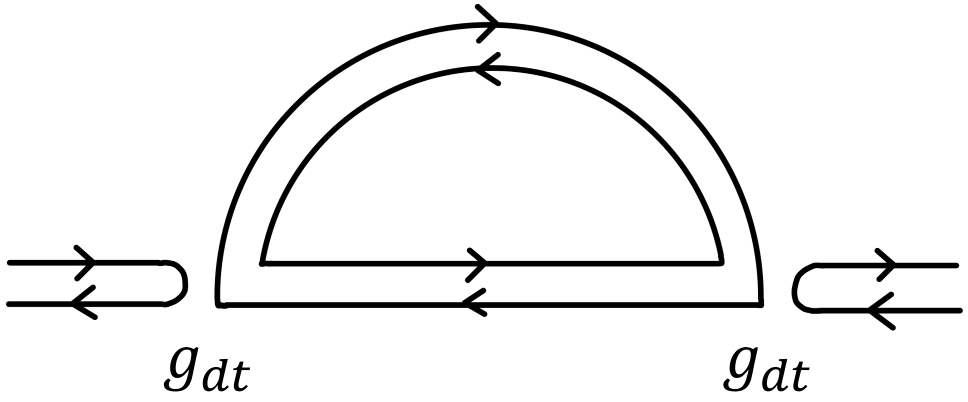
$$\frac{logZ(\vec{J}=0)}{N} = \underbrace{\frac{3}{8\lambda}\left(1-\sqrt{1+\frac{2}{3}\lambda}\right)^2 - \frac{1}{2}\log\left(\left(1+\sqrt{1+\frac{2}{3}\lambda}\right)/2\right)}_{bubble\ diagram} + O\left(\frac{1}{N}\right)$$

4. 補遺

2.2 $Matrix\ model$

$$\mathcal{L}_{int}^{dt} = -\frac{g_{dt}}{3!} tr(\Phi^2) \times tr(\Phi)$$





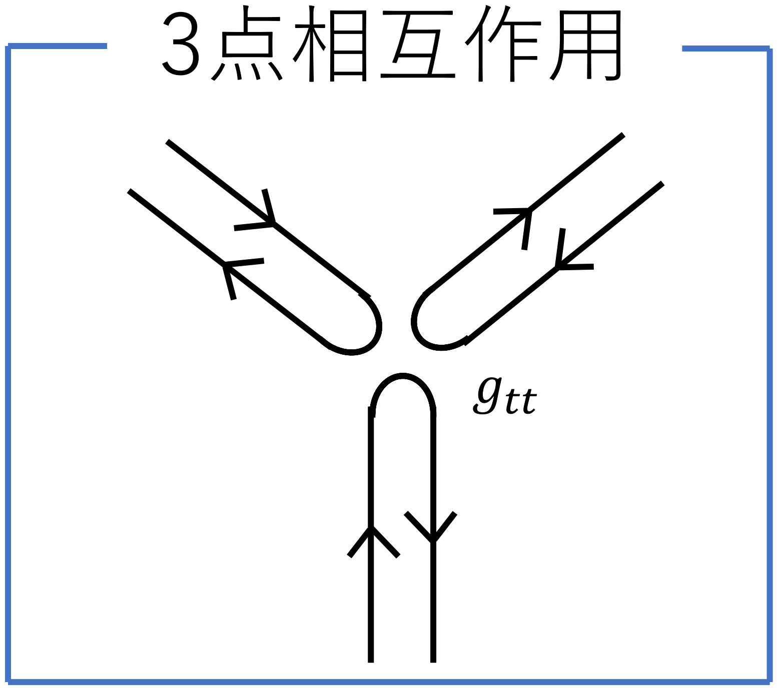
$$\lambda_{dt} = (g_{dt})^2 N^2$$

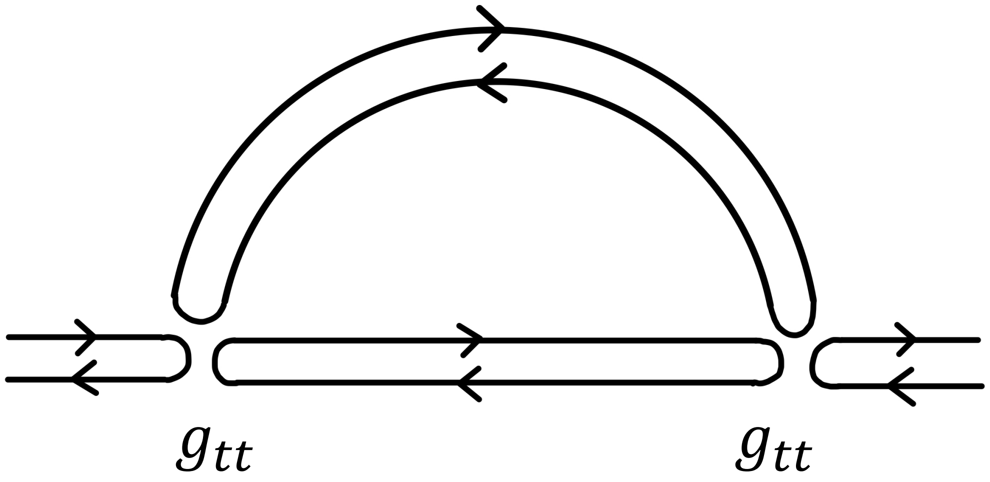
$$g_{dt} = \frac{\sqrt{\lambda_{dt}}}{N} \ll g = \frac{\sqrt{\lambda}}{\sqrt{N}}$$

4. 補遺

2.2 $Matrix\ model$

$$\mathcal{L}_{int}^{tt} = -\frac{g_{tt}}{3!} tr(\Phi) \times tr(\Phi) \times tr(\Phi)$$





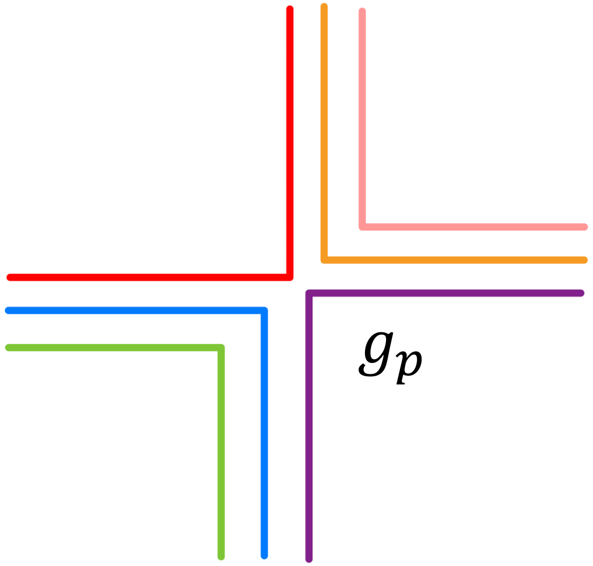
$$\lambda_{tt} = (g_{tt})^2 N^2$$

$$g_{tt} = \frac{\sqrt{\lambda_{tt}}}{N} \ll g = \frac{\sqrt{\lambda}}{\sqrt{N}}$$

4. 補遺

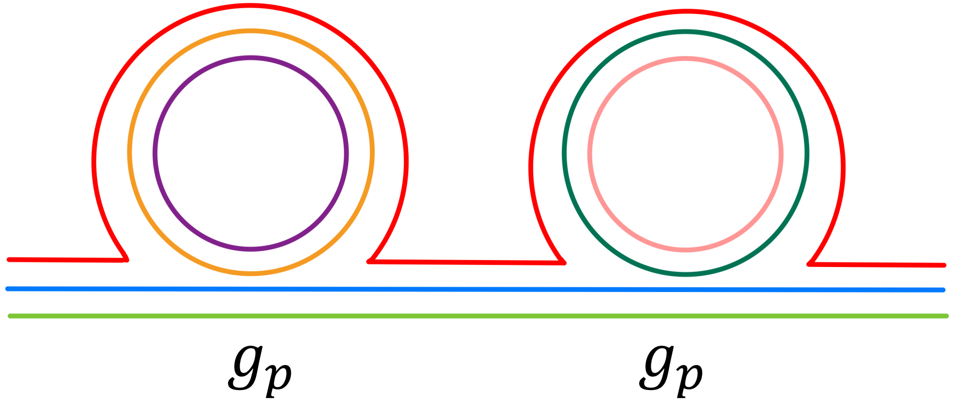
$2.3\ Tensor\ model$

$$\mathcal{L}_{int}^{p} = -\frac{g_p}{4!}\phi^{i_1 j_1 k_1}\phi^{i_2 j_1 k_1}\ \phi^{i_2 j_2 k_2}\phi^{i_1 j_2 k_2}$$

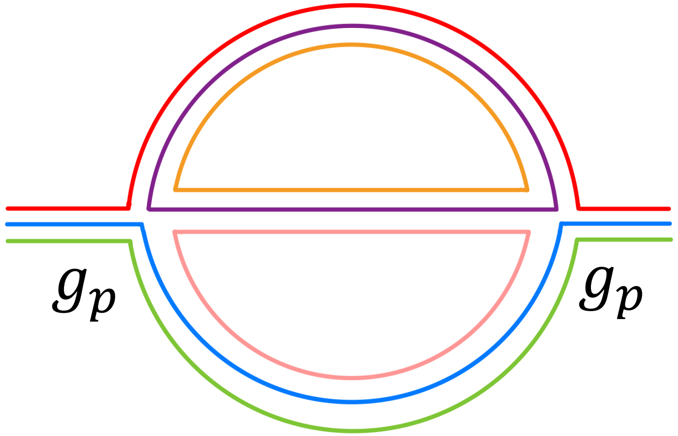


$$\lambda_p = g_p N^2$$

$$g_p = \frac{\lambda_p}{N^2} \ll g = \frac{\lambda}{N^{3/2}}$$



$$g_p^2 N^4 = \lambda_p^2$$

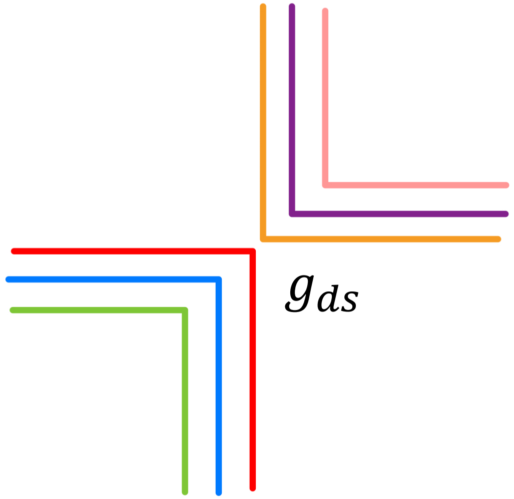


$$g_p^2 N^3 = \lambda_p^2 / N$$

4. 補遺

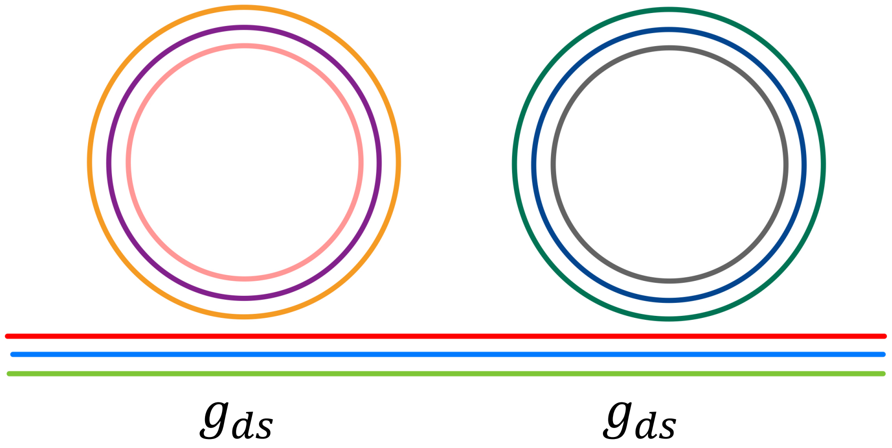
2.3 *Tensor model*

$$\mathcal{L}_{int}^{ds} = -\frac{g^{ds}}{4!}\left(\phi^{i_1 j_1 k_1}\phi^{i_1 j_1 k_1}\right)^2$$

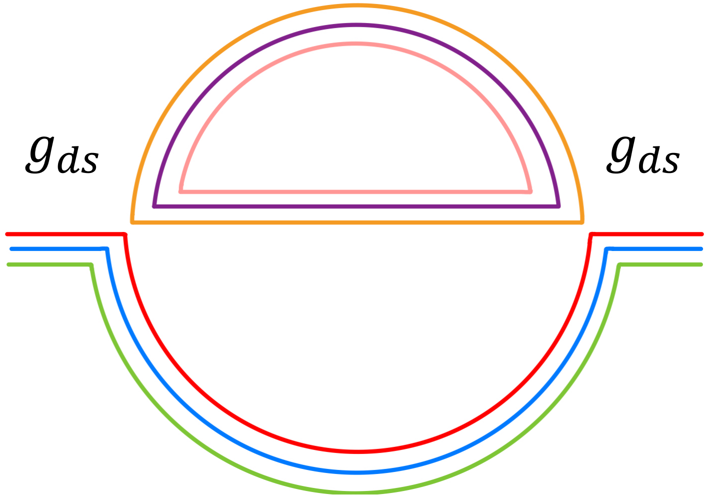


$$\lambda_{ds} = g_{ds}N^3$$

$$g_{ds} = \frac{\lambda_{ds}}{N^3} \ll g = \frac{\lambda}{N^{3/2}}$$



$$g_{ds}^2 N^6 = \lambda_{ds}^2$$



$$g_{ds}^2 N^3 = \lambda_{ds}^2 / N^3$$