Seminar@University of Tsukuba

Quantum Mpemba Effect in CFT and Quantum Simulation

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大阪大学
THE UNIVERSITY OF OSAKA

My HP

2026/7/24 14:00-15:00

Based on arXiv:2509.05597 and ongoing work.

1. Introduction

Today's topic

Nonequilibrium physics

Conformal Field Theories(CFT)

Quantum Computing

1. Introduction

Today's topic

Mpemba effect

Anomalous relaxation in nonequilibrium physics

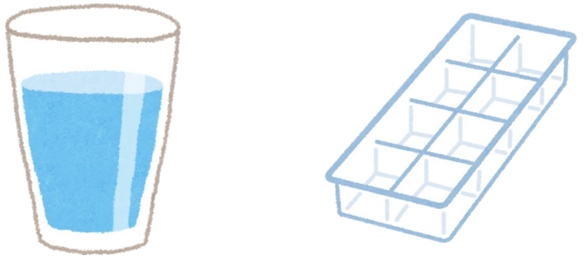
What is the Mpemba effect?

Conformal Field Theories(CFT)

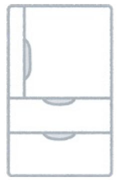
Quantum Computing

1. Introduction

Consider the following question: How can we make ice faster?



Cold water

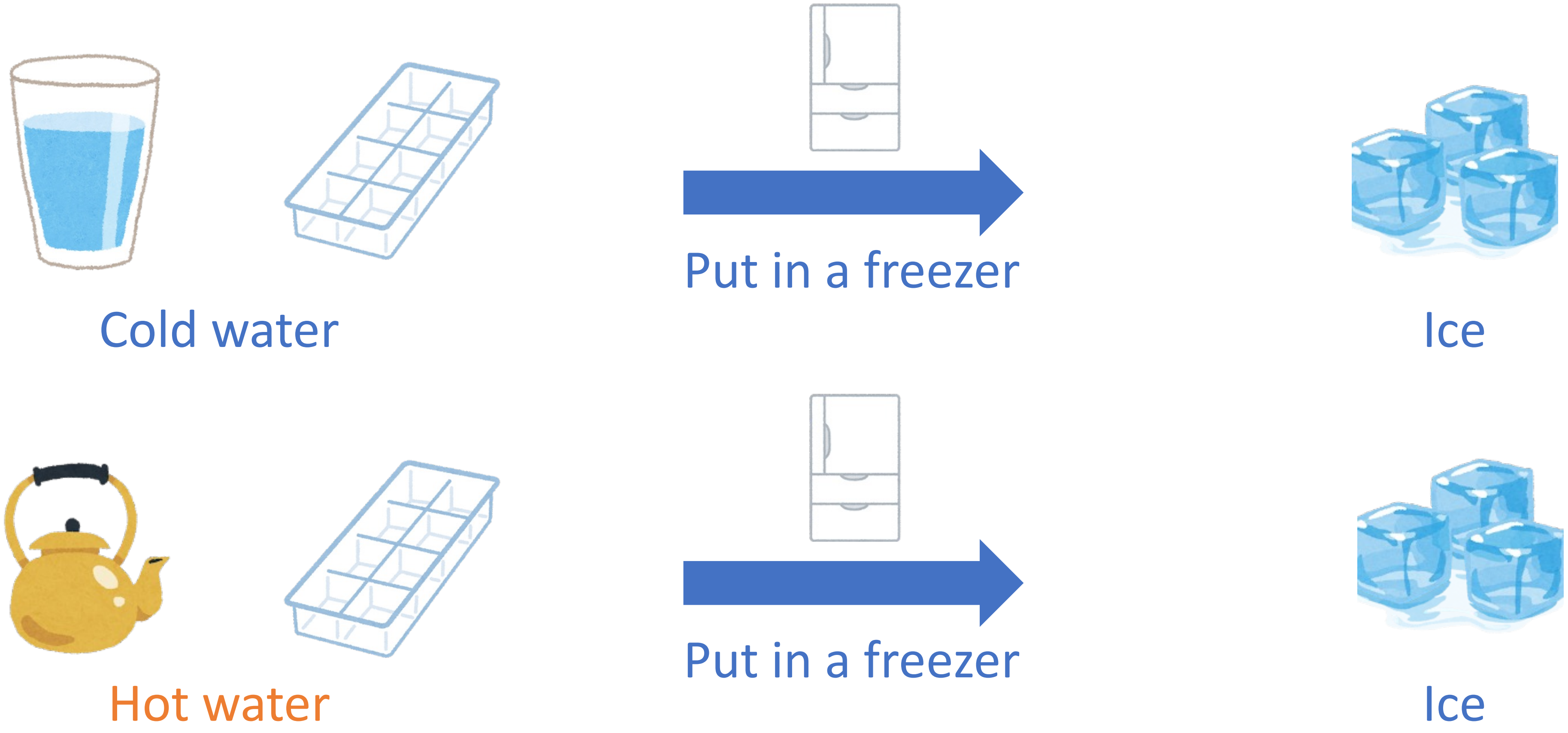


Put in a freezer

Ice

1. Introduction

Consider the following question: How can we make ice faster?

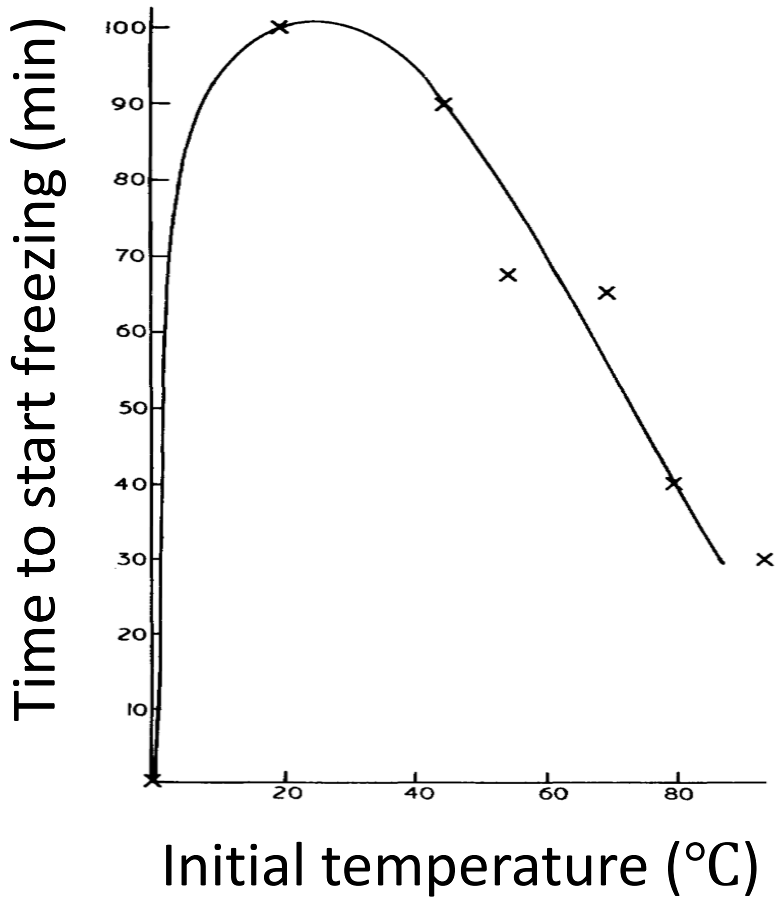


Intuitively, one expect cold water to freeze faster than hot water.

1. Introduction

Mpemba and Osborne conduct an experiment and a surprising phenomenon was observed.

[E B Mpemba and D G Osborn, 1969]



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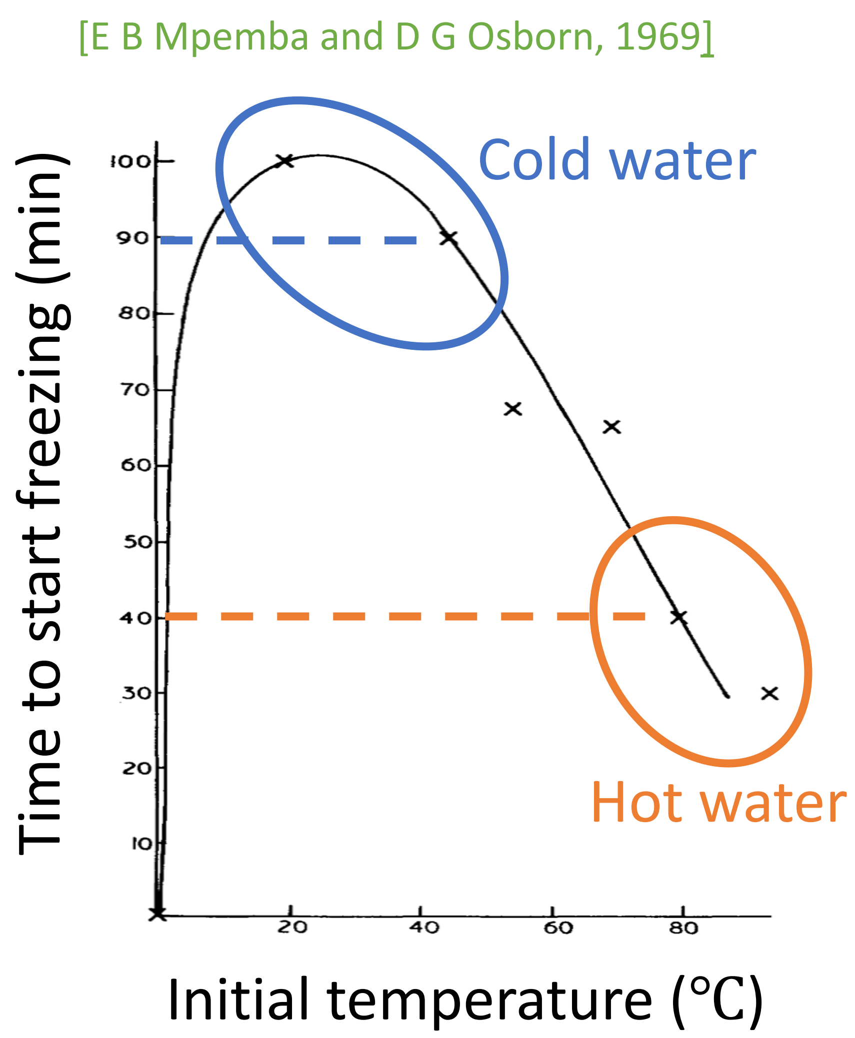


Hot water can freeze faster than cold water!

1. Introduction

Question:
How can we make ice faster?

Answer:
Use hot water instead of cold water.

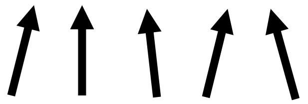
Source: [Science-tech/2023-03-25]

This is the Mpemba effect:
an anomalous relaxation in nonequilibrium physics

1. Introduction

The Mpemba effect was first observed in classical systems, but a quantum counterpart has been found.
[Ares-Murciano-Calabrese, 2022]

Symmetry-broken state

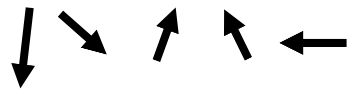


Initial state 1

(Weakly symmetry-broken)

Quench with a symmetric Hamiltonian

Symmetry restoration



(symmetric state)

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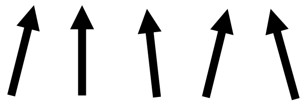

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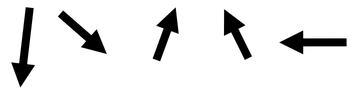
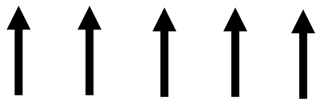
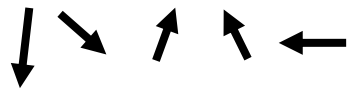


(symmetric state)



Initial state 2

(Strongly symmetry-broken)

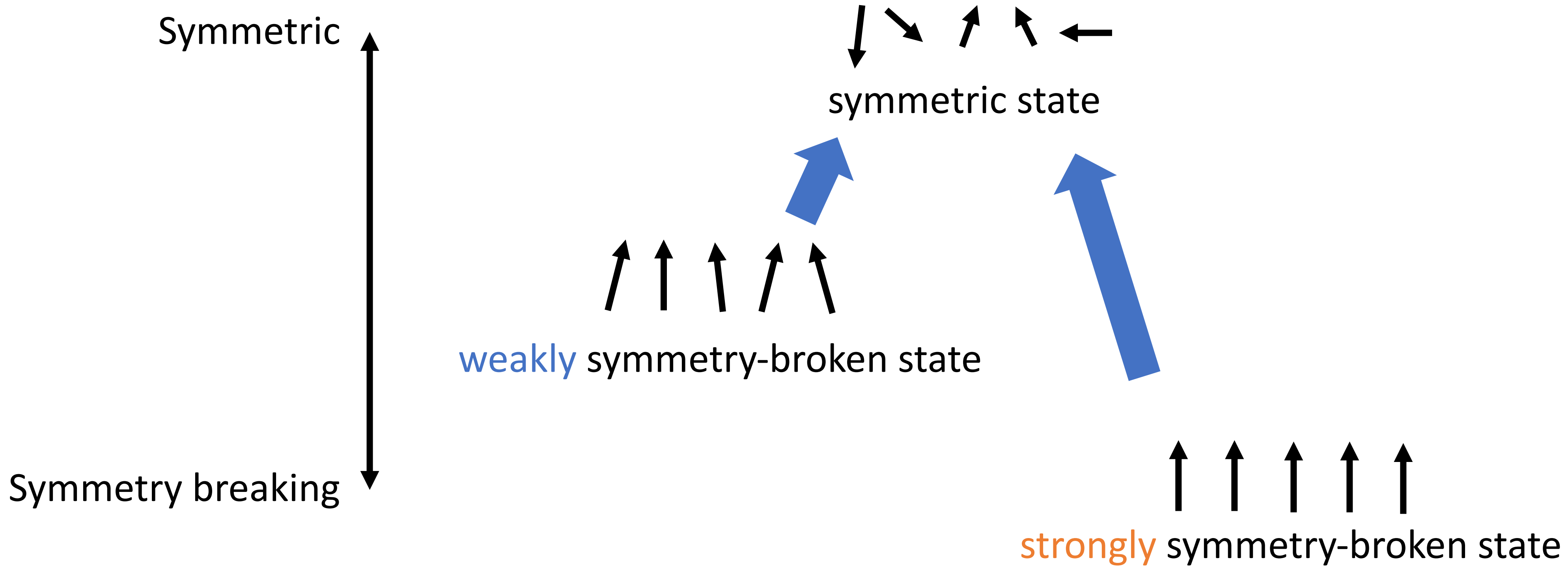


(symmetric state)

Question: Which state relaxes faster to the symmetric state?

1. Introduction

Intuitively, one expects the weakly symmetry-broken state to relax faster, since it is initially closer to the symmetric state.



Surprisingly, the opposite can happen.

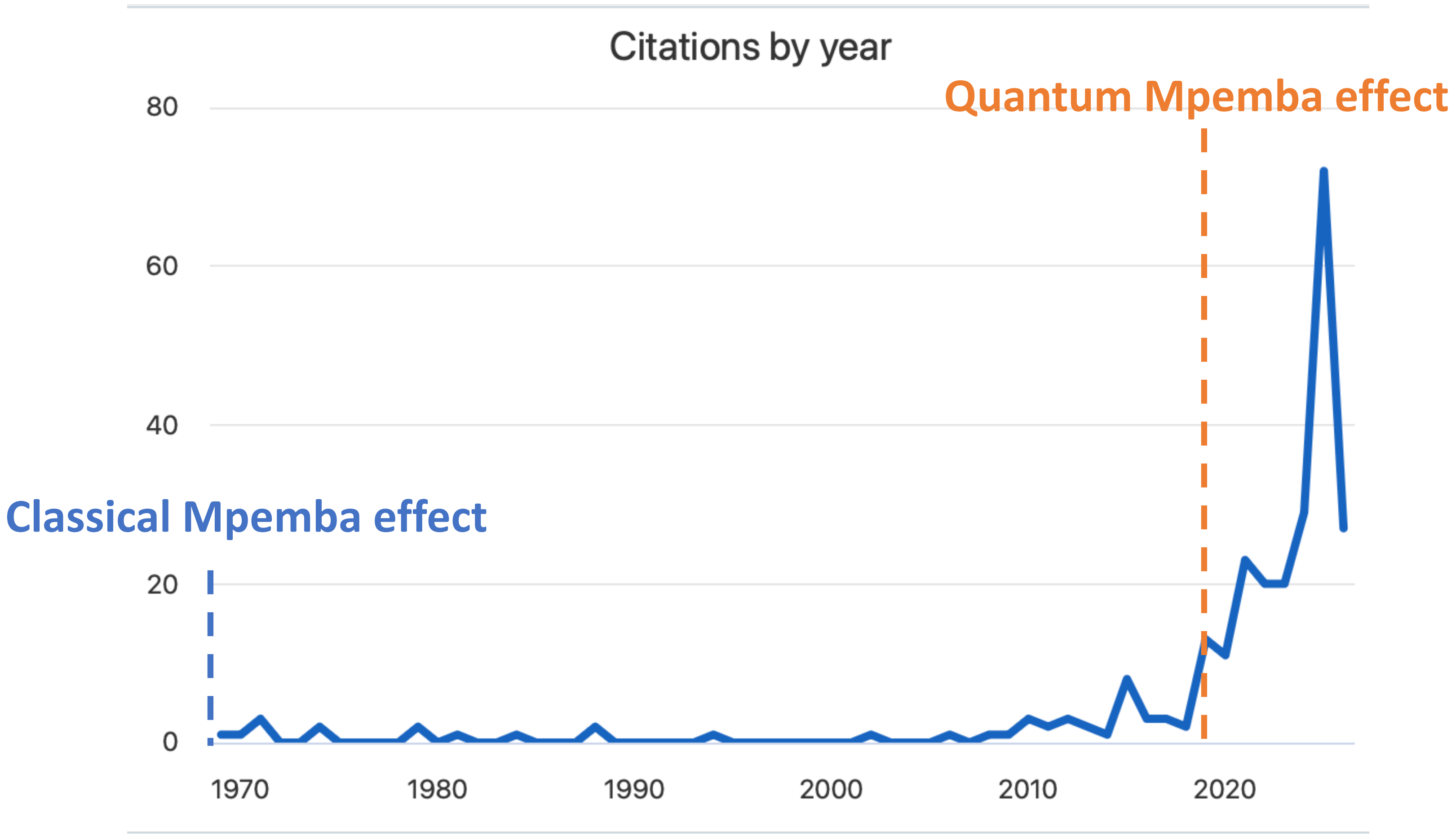
Counterintuitive!

Quantum Mpemba effect

The more symmetry is initially broken, the faster it is restored.

1. Introduction

The Mpemba effect has attracted considerable attention.



Research on the Mpemba effect has grown rapidly!

1. Introduction

Previous studies

- ☑ Quantum spin systems (one and two spatial dimensions; integrable systems)
  [Murciano et al., 2023], among others
- ☑ Experimental observations [Joshi et al., 2024], among others
- ☑ Control and engineering applications
- ✗ Few results are available in quantum field theory.
  [Benini-Godet-Singh, 2024]
  Existing QFT studies are limited to abelian symmetry in (1+1)-dimensional CFTs.

The quantum Mpemba effect for non-Abelian symmetries remains poorly understood.

It is challenging to analyze the quantum Mpemba effect in quantum field theory.

1. Introduction

Overview of this talk

Mpemba effect
Anomalous relaxation in nonequilibrium physics

Conformal Field Theories(CFT)

Quantum Mpemba effect for non-abelian symmetry

Quantum Computing

Quantum algorithm to study quantum Mpemba effect

1. Introduction

Overview of this talk

Mpemba effect

Anomalous relaxation in nonequilibrium physics

Main part
2509.05597

Ongoing work

Conformal Field Theories(CFT)

Quantum Mpemba effect for non-abelian symmetry

Quantum Computing

Quantum algorithm to study quantum Mpemba effect

1. Introduction

Short summary(CFT)

- We study the quantum Mpemba effect in Wess–Zumino–Witten models using CFT techniques.
- We analytically establish the quantum Mpemba effect for non-Abelian symmetries in QFT for the first time.
- Moreover, we uncover a new type of quantum Mpemba effect (explained later).

(Ongoing work)

- We develop a quantum algorithm for simulating the quantum Mpemba effect in QFT.
- We apply it to the lattice Schwinger model and demonstrate the quantum Mpemba effect.

Outline

1. Introduction
2. Entanglement Asymmetry
3. CFT approach
4. Numerical approach using quantum computing (ongoing)
5. Summary

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2. Entanglement Asymmetry

Question: How can we study the quantum Mpemba effect?

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Conventionally, an order parameter $\langle \mathcal{O} \rangle$ is used to characterize symmetry breaking.

However, to study the quantum Mpemba effect, we need a **new "quantifier"** satisfying the following requirements.

Requirements for a quantifier of the quantum Mpemba effect

1. Quantifies the degree of symmetry breaking
2. Well-defined for out-of-equilibrium dynamics
3. Computable

2. Entanglement Asymmetry

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A new quantifier: Entanglement Asymmetry

[Ares-Murciano-Calabrese, 2022]

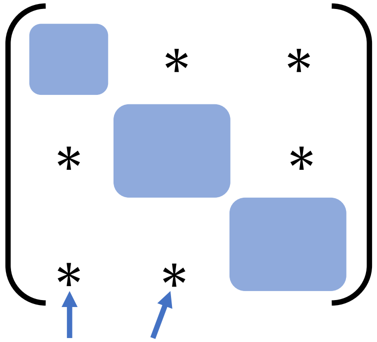
2. Entanglement Asymmetry

We assume that the theory has a symmetry with charge

$$Q = Q_A + Q_B$$

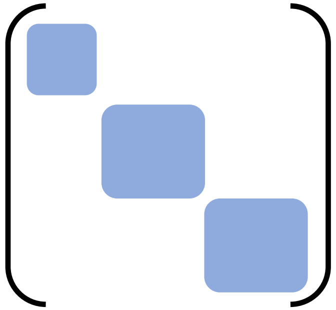
$A =$ Subsystem of interest, $\quad B =$ Environment.

Symmetry-broken state

$\rho_A =$ 

Off-diagonal elements in eigen basis of Q_A

Symmetric state

$\rho_{A,S} =$ 

Block diagonal

Entanglement Asymmetry: a quantifier of symmetry breaking

[Ares-Murciano-Calabrese, 2022]

$$\Delta S_A \equiv \Delta S(\rho_A | \rho_{A,S}) = \mathrm{Tr}_A[\rho_A(\log \rho_A - \log \rho_{A,S})]$$

Relative entropy

2. Entanglement Asymmetry

Key properties of entanglement asymmetry:

1. Defined for any state, including nonequilibrium states.

2. Non-negativity:

$$\begin{cases} \Delta S_A = 0 \iff [\rho_A, Q_A] = 0 & \textbf{(symmetric)} \\ \Delta S_A > 0 \iff [\rho_A, Q_A] \neq 0 & \textbf{(symmetry broken)} \end{cases}$$

[Kullbck-Leibler, 1951]

3. Quantifies subsystem symmetry breaking:

$$0 < \underbrace{\Delta S_A}_{\text{Weak,}} < \underbrace{\Delta S'_A}_{\text{Strong}}$$

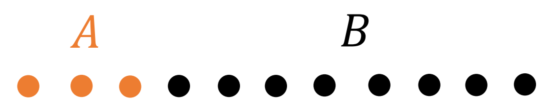
Degree of symmetry breaking = Weak, Strong

Entanglement asymmetry is a well-defined measure of symmetry breaking.

2. Entanglement Asymmetry

Typical protocol to investigate the quantum Mpemba effect.

Quench dynamics with a symmetric Hamiltonian



Spin chain or QFT

Total system=$A \cup B$

A : subsystem of interest

STEP1: Prepare an initial state $|\psi_{AB}(0)\rangle$ that **explicitly** breaks the symmetry.

STEP2: Perform the time evolution.

$$|\psi_{AB}(t)\rangle = e^{-iHt}|\psi_{AB}(0)\rangle,$$ where H is a symmetric Hamiltonian.

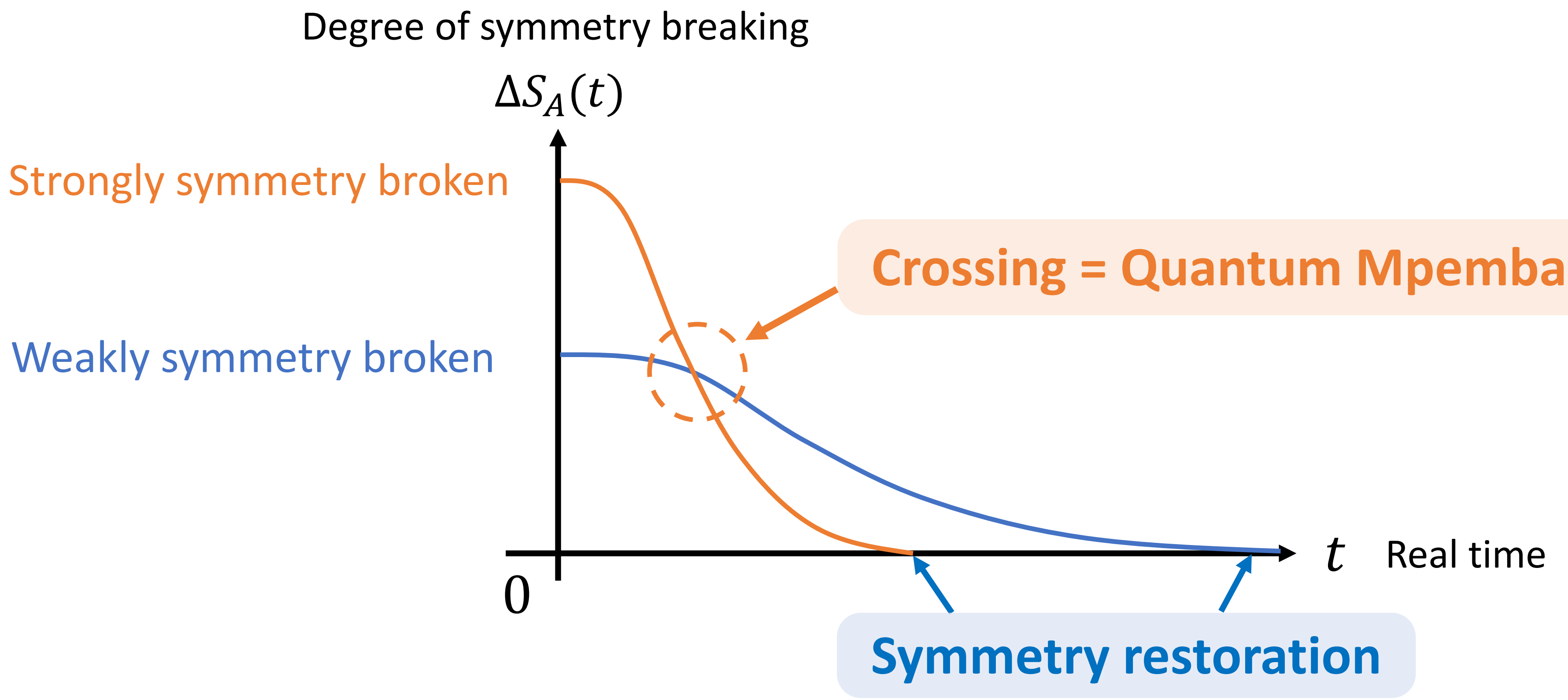
STEP3: Compute the entanglement asymmetry at time t.

$$\Delta S_A(t)$$

Entanglement asymmetry captures the real-time dynamics of symmetry restoration.

2. Entanglement Asymmetry

Typical behavior of the quantum Mpemba effect.



Using this protocol, the quantum Mpemba effect has been studied extensively in condensed-matter physics, high-energy physics, and quantum information.

Outline

1. Introduction

2. Entanglement Asymmetry

3. CFT approach [HF, S.Shimamori 2025]

4. Numerical approach using quantum computing (ongoing)

5. Summary

3. CFT approach

Question: How can entanglement asymmetry be formulated in QFT?

Assumptions

- The theory has a global symmetry G
- The Hilbert space factorizes: $\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$

A: subsystem, B: environment

$$U_{tot}(g) = U_A(g) \otimes U_B(g), \;\; g \in G$$

Factorized action of the symmetry operator

Symmetry-broken state

$$\rho_A = \mathrm{Tr}_A[|\psi\rangle\langle\psi|]$$

$$[\rho_A, U_A] \neq 0$$

Symmetric state

$$\rho_{A,S} = \int_G \underbrace{\mathrm{d}g}_{\text{Haar measure}} \, U_A(g)\rho_A U_A^\dagger(g)$$

$[\rho_{A,S}, U_A(g)] = 0$. for any $g \in G$

3. CFT approach

Question: How can entanglement asymmetry be formulated in QFT?

Entanglement Asymmetry

$$\Delta S_A \equiv \Delta S(\rho_A|\rho_{A,S}) = \mathrm{Tr}_A[\rho_A(\log\rho_A - \log\rho_{A,S})]$$

Relative entropy

Difficult to access…

Rényi Entanglement Asymmetry (REA)

$$\Delta S_A^{(n)} \equiv \frac{1}{1-n}\log\frac{\mathrm{Tr}_A[\rho_{A,S}^n]}{\mathrm{Tr}_A[\rho_A^n]}, \quad \lim_{n\to1}\Delta S_A^{(n)} = \Delta S_A$$

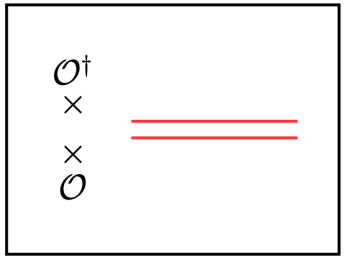
Computable!

3. CFT approach

Let me explain how we can investigate the Rényi entanglement asymmetry in CFT.

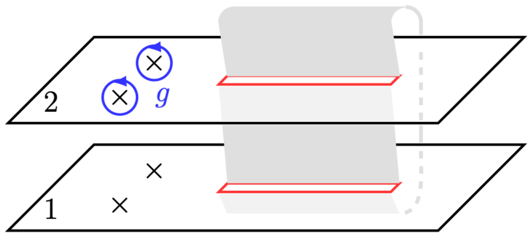
Flow of analysis

STEP 1
Path integral rep

$\rho_A =$ 

STEP 2
Replica trick

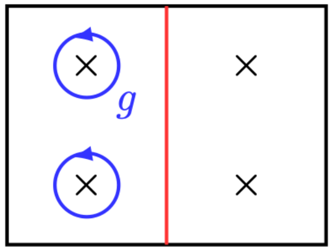
[Benini et al, 2024]



$\mathcal{R}_n$

STEP 3
Conformal map

[He et al, 2024]

$\mathcal{R}_n \to$ 

$\mathbb{R}^2$

3. CFT approach

STEP 1: Path integral representation

Let us consider (1+1)-dimensional CFT with a continuous global symmetry G.

By the Mermin-Wagner theorem, the vacuum state $|0\rangle$ does not exhibit spontaneous symmetry breaking.

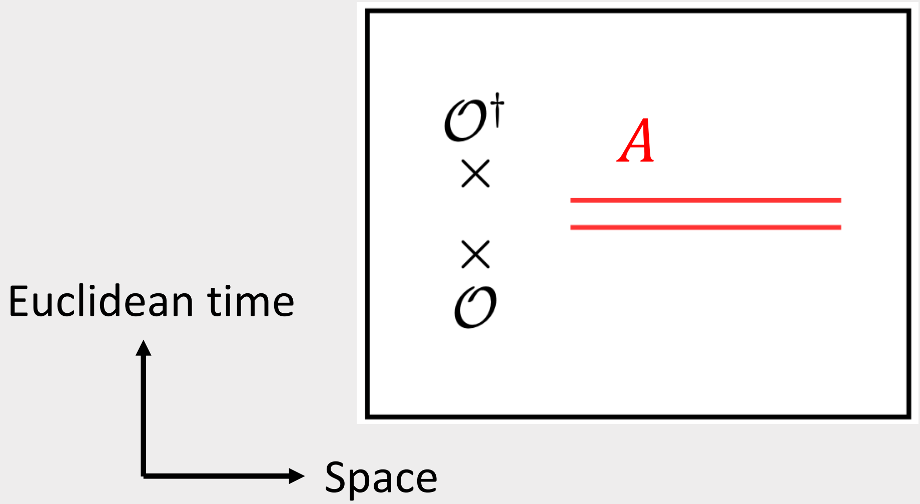
[Mermin-Wagner, 1966]

We explicitly break the symmetry through a local operator insertion $\mathcal{O}$.

$|\psi\rangle = \mathcal{O}|0\rangle$: **explicitly** symmetry-broken state

Path integral representation

$$\rho_A = \mathrm{Tr}_{\mathrm{A}}[|\psi\rangle\langle\psi|] =$$



3. CFT approach

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[Mermin-Wagner, 1966]

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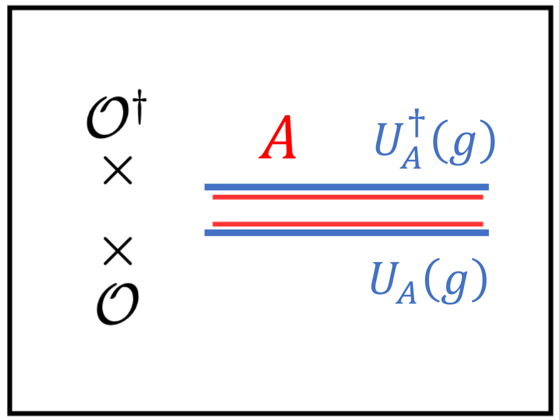
Path integral representation

$$\rho_{A,S} = \int_G \mathrm{d}g\, U_A(g)\rho_A U_A^\dagger(g) =$$

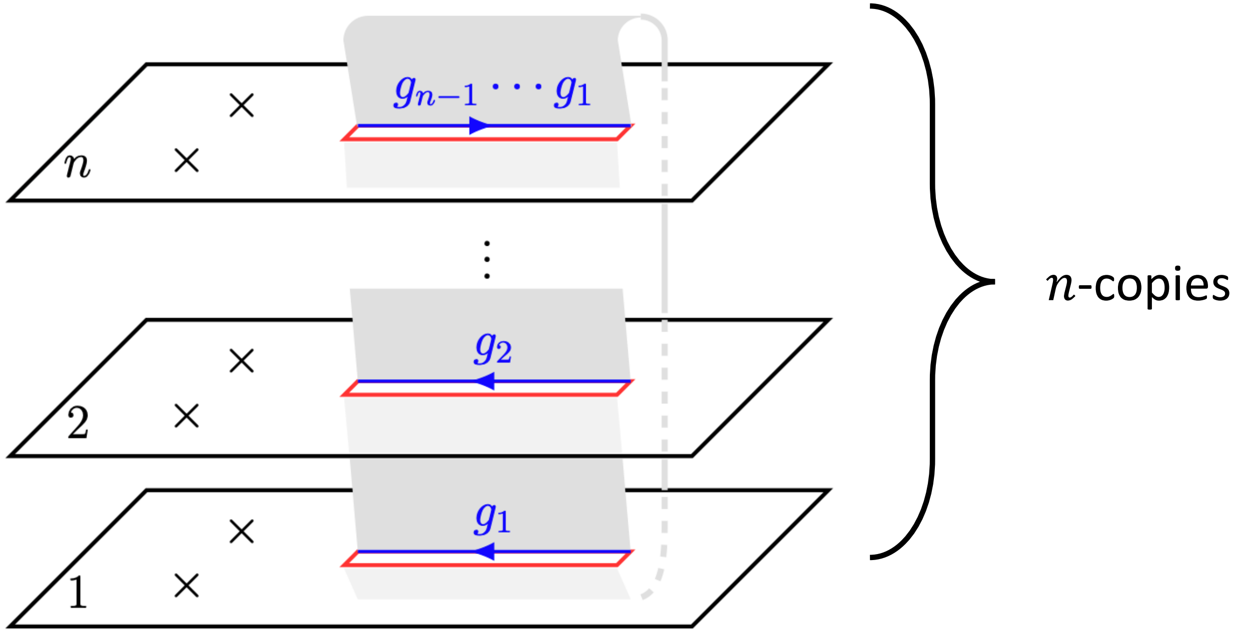


STEP 2: Replica trick

To compute the relevant moments, we replicate spacetime.

For example, $\mathrm{Tr}_A\left[\rho_{A,S}^n\right]$ can be schematically written as follows.

$$\mathrm{Tr}_A\left[\rho_{A,S}^n\right] =$$



$\mathcal{R}_n$

Replicated manifold

3. CFT approach

STEP 2: Replica trick

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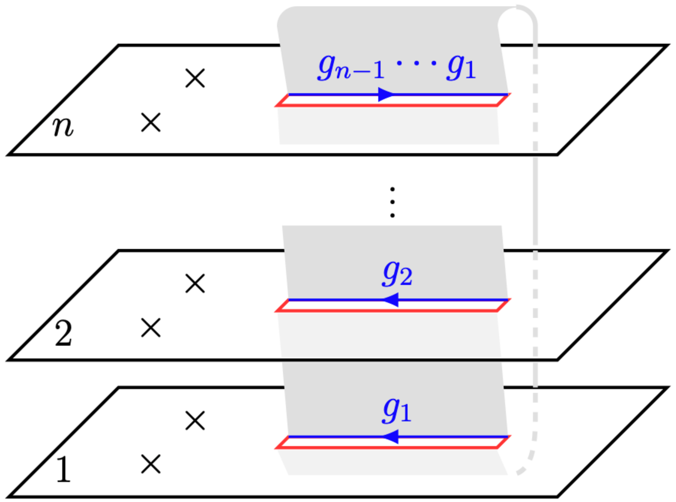
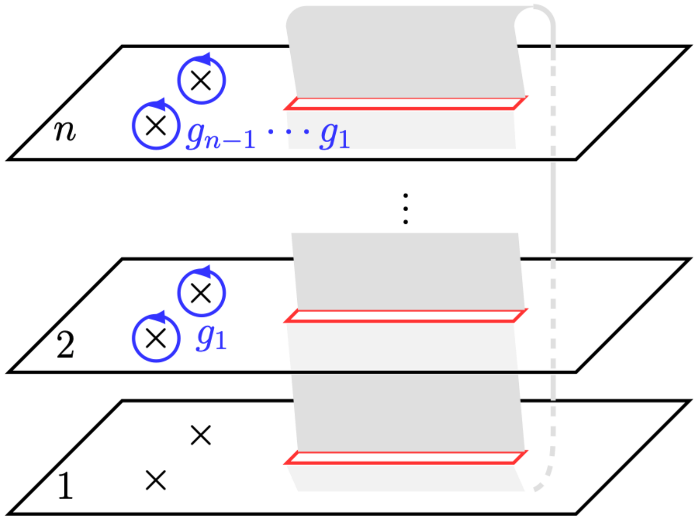
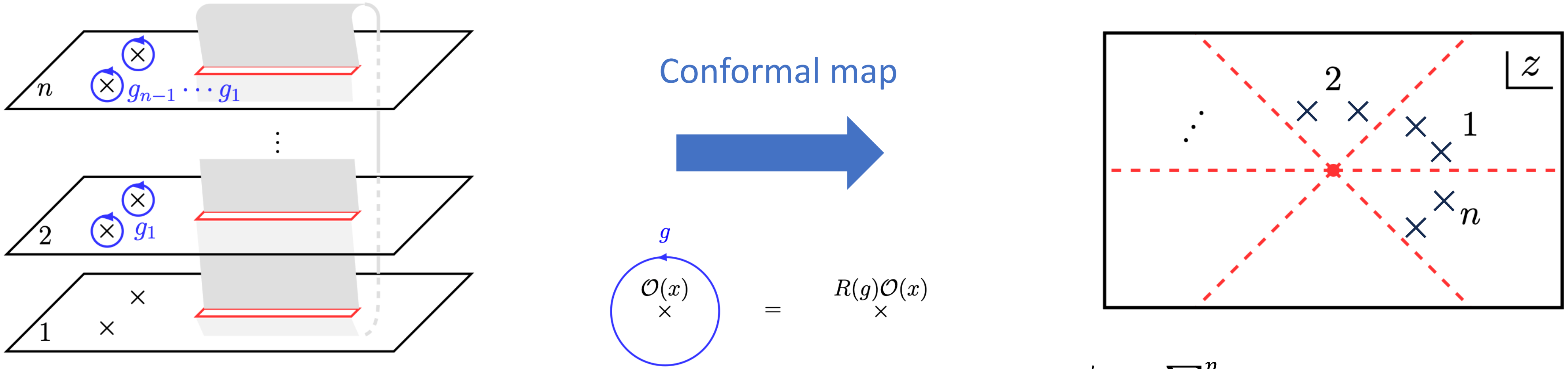


Topological property of symmetry operators



STEP 3: Conformal map

In a (1+1)-dimensional CFT, we can use the following conformal map. [He et al, 2024]



$$\sim \left\langle \mathcal{O}_1\mathcal{O}_1^\dagger \sum_{i=2}^{n} [R(g_{i-1}\cdots g_1)\mathcal{O}]_i [\bar{R}(g_{i-1}\cdots g_1)\mathcal{O}^\dagger]_i \right\rangle_{\text{Plane}}$$

Correlation function on Plane!

3. CFT approach

In summary, conformal symmetry and the properties of symmetry operators drastically simplified the calculation.

n-th Rényi entanglement asymmetry

$$\Delta S_A^{(n)}$$

A measure of symmetry breaking

$2n$-point function

$$\left\langle \mathcal{O}_1 \mathcal{O}_1^\dagger \sum_{i=2}^{n} [R(g_{i-1} \cdots g_1)\mathcal{O}]_i [\bar{R}(g_{i-1} \cdots g_1)\mathcal{O}^\dagger]_i \right\rangle_{\text{Plane}}$$

Rényi entanglement asymmetry reduces to correlation functions on Plane!

3. CFT approach

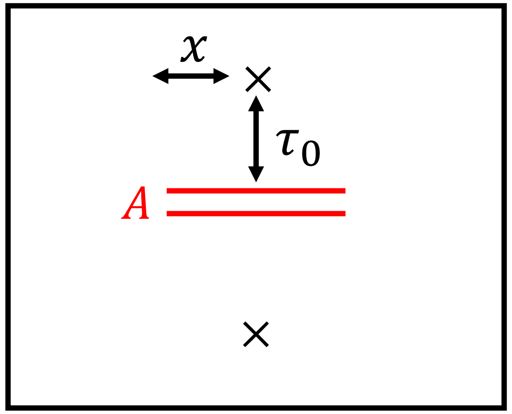
Our concrete analysis

- Model: $\widehat{su}(N)_k$ Wess-Zumino-Witten model
- Initial state: $|\psi\ (t=0)\rangle = \Phi_i(x,\tau_0)|0\rangle$

  Φ_i : primary field in the fundamental representation $(i = 1, \ldots, N)$

  Explicit breaking of the $SU(N)$ symmetry

- Target: 2nd Rényi entanglement asymmetry $\Delta S_A^{(2)}$

$\rho_A =$ 

Technical remarks

- We introduce a parameter τ_0 that controls the degree of initial symmetry breaking.
- Real time t is implemented through analytic continuation. [S. He, et al 2014]
- The 4-point function of the WZW model is known explicictely. [Knizhnik and Zamolodchikov, 1984]
- Thus, we obtain a closed-form expression of the $\Delta S_A^{(2)}$ (although it is lengthy).

3. CFT approach

Our result

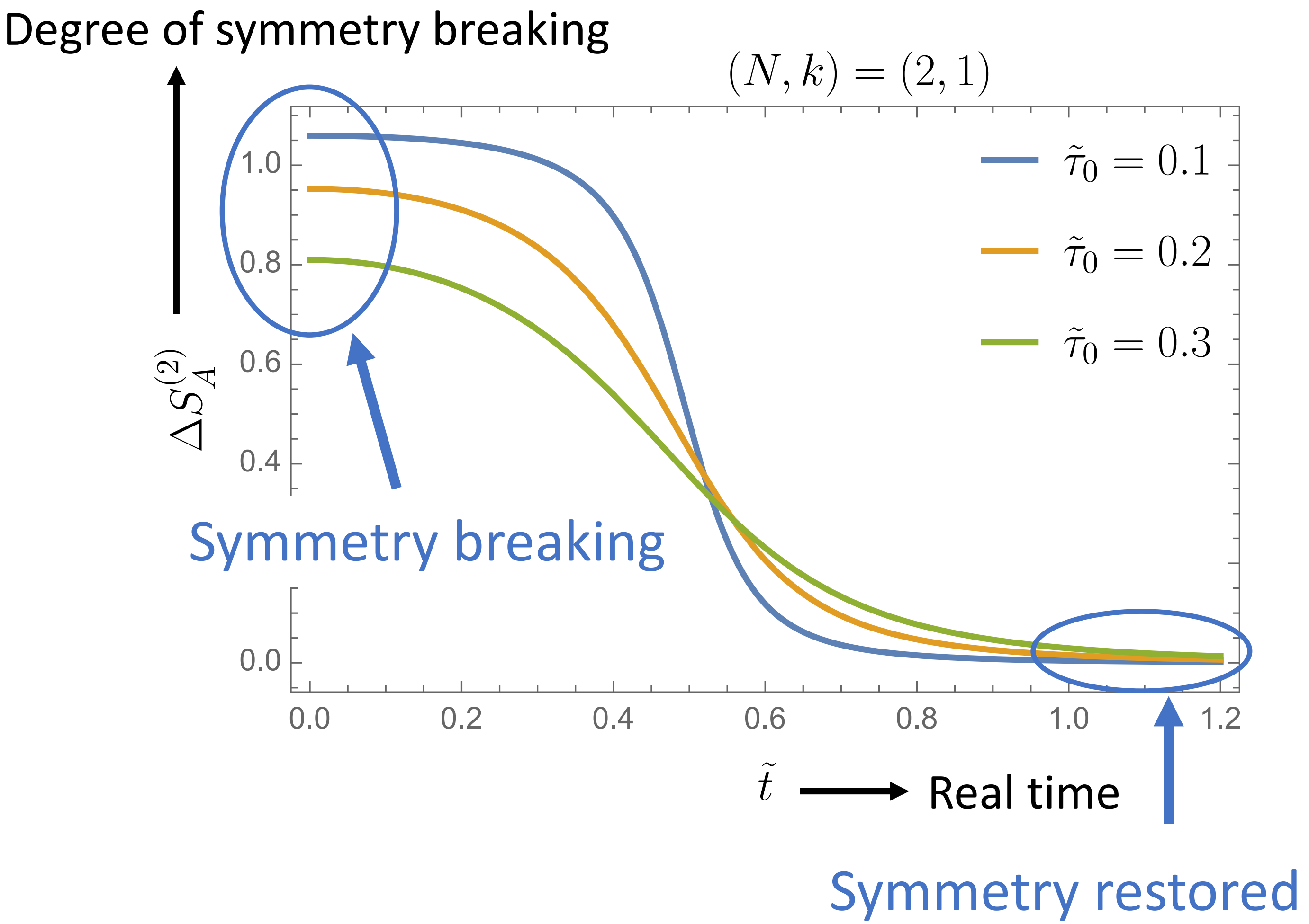
Initial state

$$|\psi\ (t=0)\rangle = \Phi_i(x, \tau_0)|0\rangle$$

τ_0: parameter of initial symmetry breaking.

$i = 1, \cdots, N$: index of fundamental rep of $SU(N)$

Change τ_0, while fixing N



3. CFT approach

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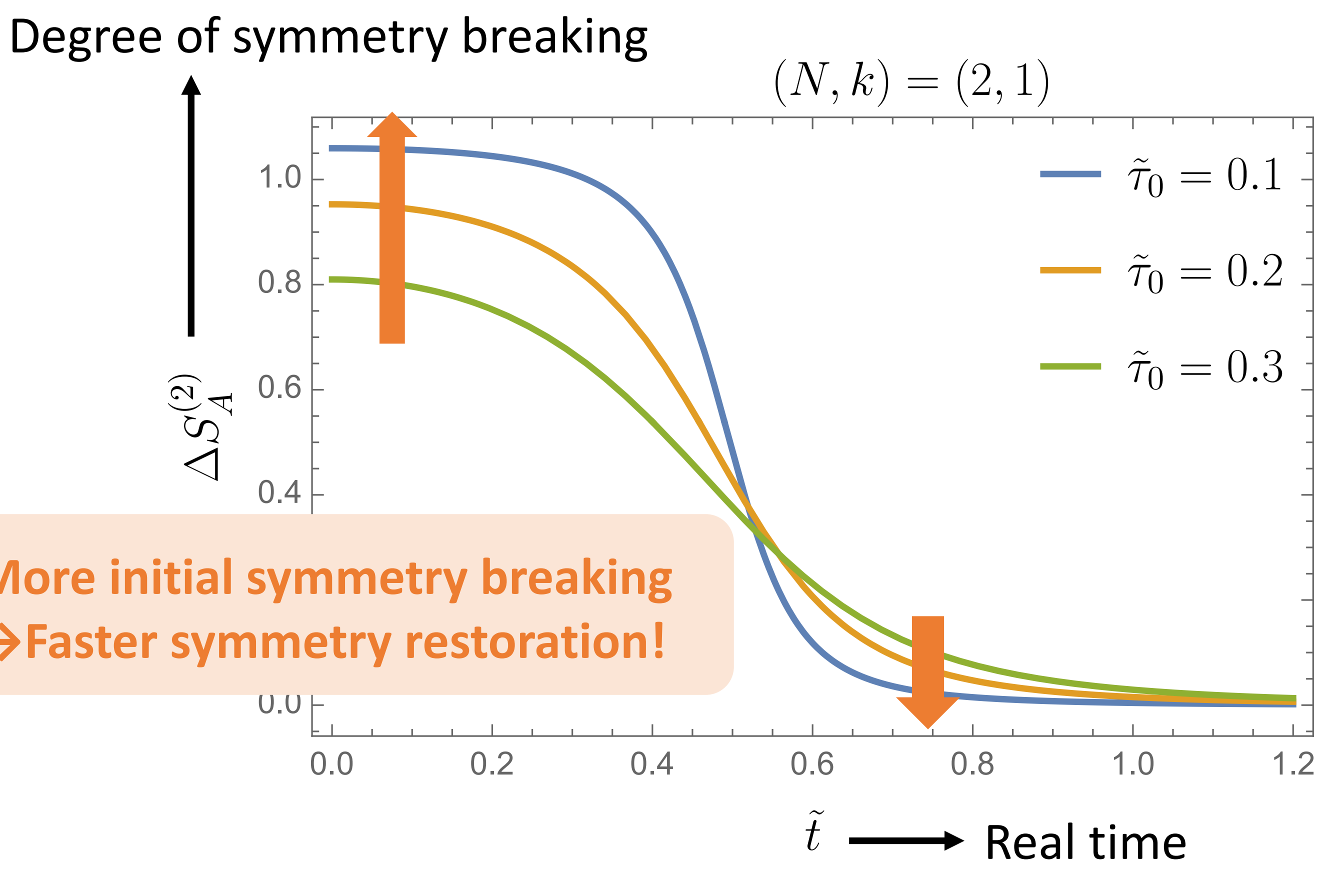


More initial symmetry breaking
→Faster symmetry restoration!

We analytically establish the quantum Mpemba effect for a non-Abelian symmetry.

3. CFT approach

Let us see **the rank dependence**

Initial state

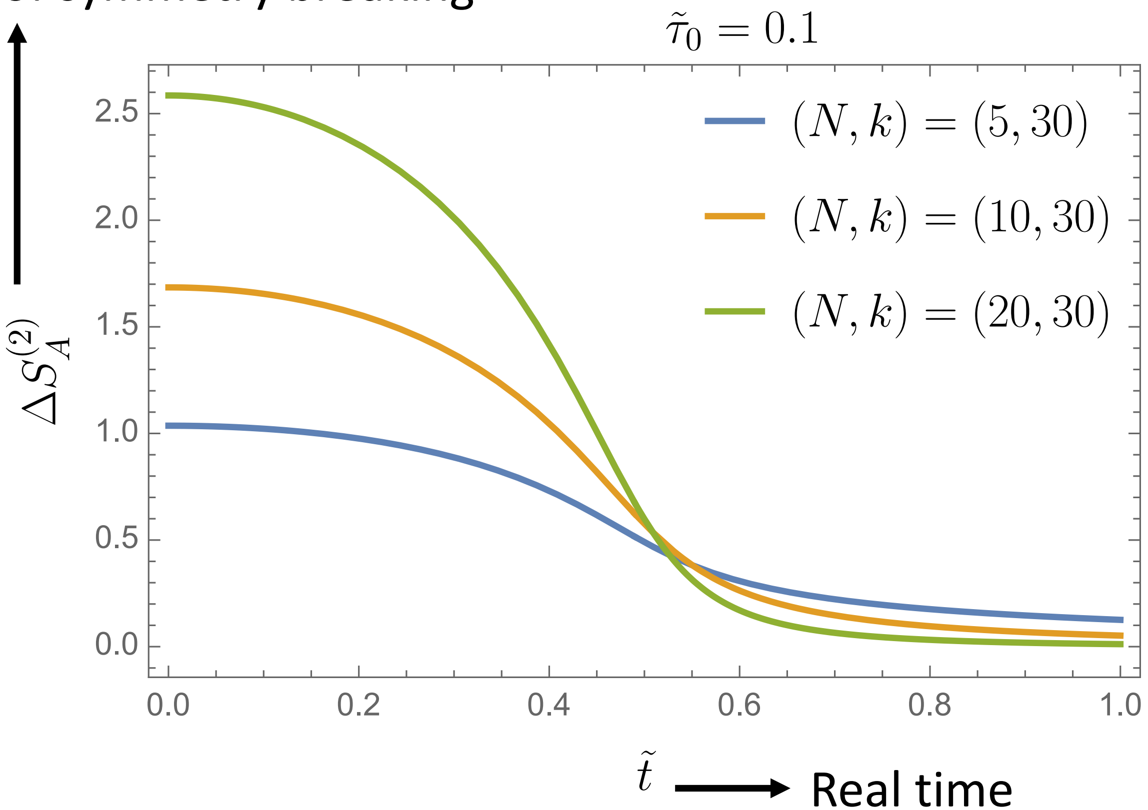
$$|\psi\,(t=0)\rangle = \Phi_i(x,\tau_0)|0\rangle$$

τ_0: parameter of initial symmetry breaking.

$i = 1, \cdots, N$: index of fundamental rep of $SU(N)$

Change N, while fixing τ_0

Degree of symmetry breaking



Real time

3. CFT approach

Rank dependence

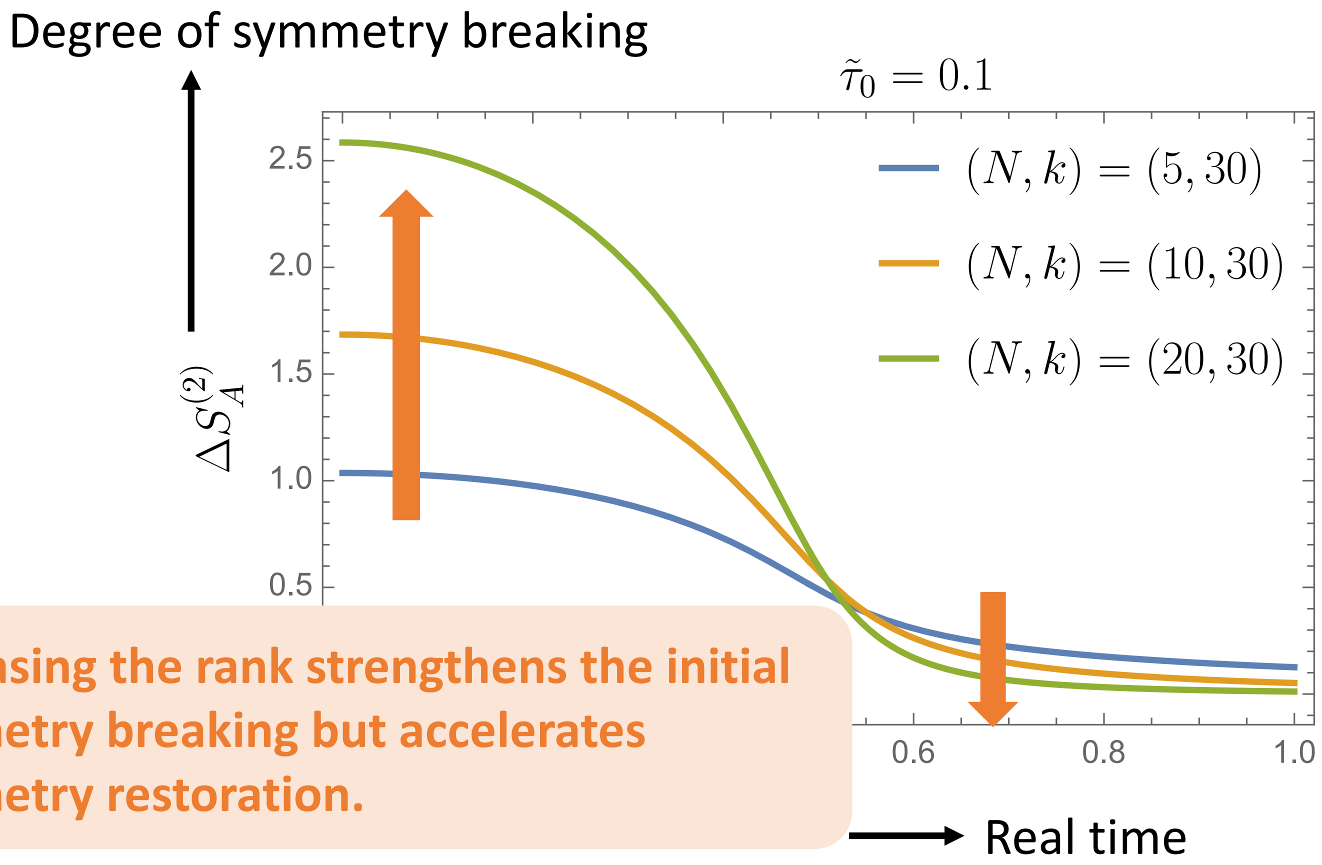
Initial state

$$|\psi\,(t=0)\rangle = \Phi_i(x,\tau_0)|0\rangle$$

τ_0: parameter of initial symmetry breaking.

$i = 1,\cdots,N$: index of fundamental rep of $SU(N)$

Change N, while fixing τ_0



Increasing the rank strengthens the initial symmetry breaking but accelerates symmetry restoration.

For example, $SU(3)$ symmetry restoration is faster than that of $SU(2)$.

We uncover new type of quantum Mpemba effect!

Outline

1. Introduction
2. Entanglement Asymmetry
3. CFT approach
4. Numerical approach using quantum computing (ongoing)

   Joint work with Keisuke Fujii, Masazumi Honda and Le Duc Truyen.

5. Summary

4. Numerical approach using quantum computing

What is the quantum computing?

Classical computing

Unit of information: bit $\boxed{0}$ or $\boxed{1}$

Data: N-bit string

$\boxed{0}\ \boxed{1}\ \boxed{0}\ \cdots = |010\cdots\rangle$

Manipulates all 2^N basis states individually.

Quantum computing

Unit of information: qubit $= \alpha|0\rangle + \beta|1\rangle$

Data: N-qubit state

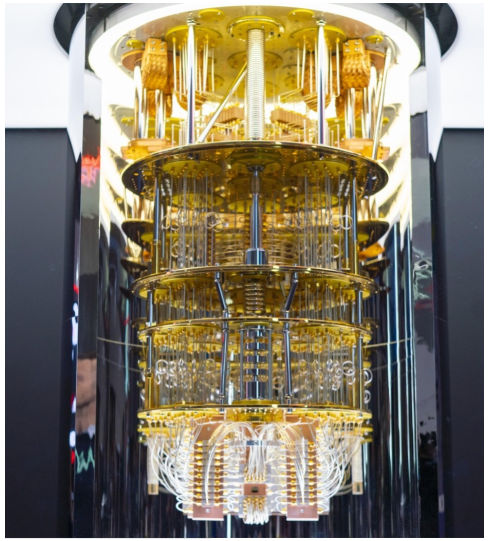
$$\cdots = \sum_{i_1,\cdots i_N=0}^{1} c_{i_1,\cdots i_N} |i_1 \cdots i_N\rangle$$

Manipulates a **superposition** of exponentially many basis states.

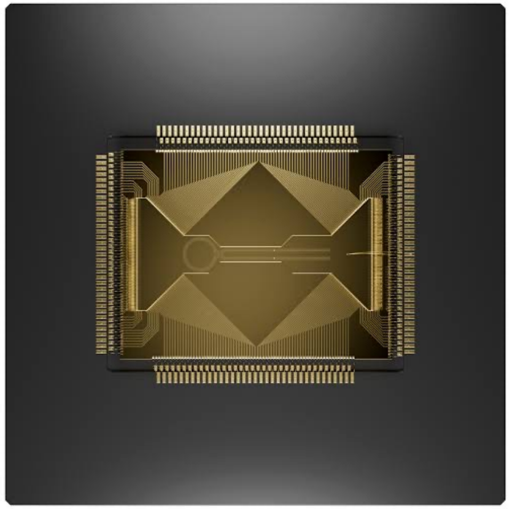
Quantum computing harnesses quantum mechanics for computation.

4. Numerical approach using quantum computing

Many research institute and industries are developing quantum computers.



IBM



Quantinuum

Riken, Fujitsu

and so on...

Quantum computers are becoming a real-world technology.

4. Numerical approach using quantum computing

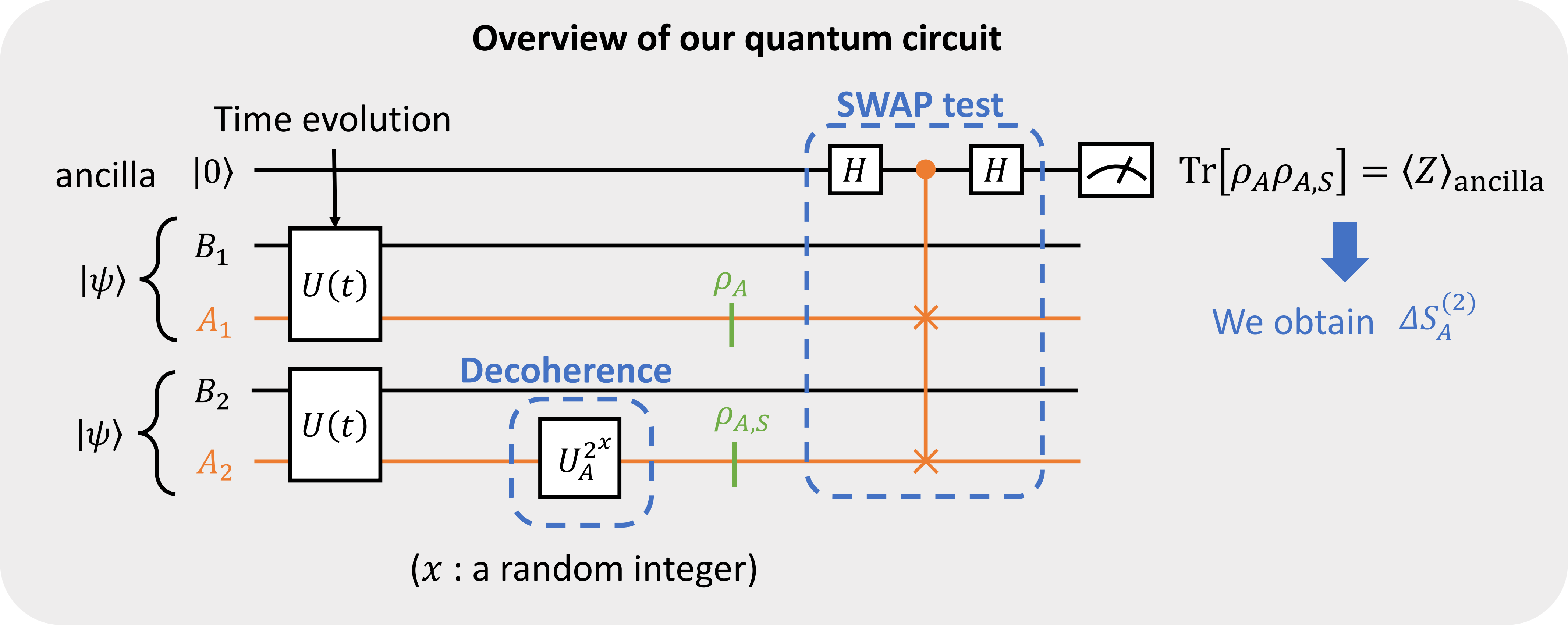
Motivations

- The CFT approach in the previous section is powerful but relies on conformal symmetry.
- For QFTs without conformal symmetry, the quantum Mpemba effect remains difficult to analyze…
  (long-time lattice simulation is required)
- On a quantum computer, real-time dynamics can be implemented directly.
- The reduced density matrix is obtained by just ignoring the environment.

↓

We develop a quantum algorithm for simulating the quantum Mpemba effect, which is suitable for QFT.

4. Numerical approach using quantum computing



By combining the idea of quantum phase estimation with SWAP test, we can estimate the Rényi Entanglement Asymmetry.

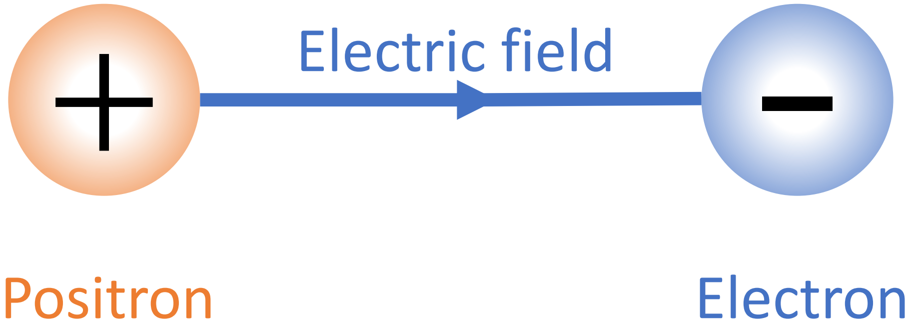
4. Numerical approach using quantum computing

Schwinger model in the continuum [Schwinger, 1962]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + \mathrm{i}gA_{\mu})\psi - m\bar{\psi}e^{\mathrm{i}\theta\gamma^5}\psi$$

||

The simplest model of quantum electrodynamics in (1+1) dimensions



4. Numerical approach using quantum computing

Schwinger model in the continuum [Schwinger, 1962]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + \mathrm{i}gA_{\mu})\psi - m\bar{\psi}e^{\mathrm{i}\theta\gamma^5}\psi$$

Temporal gauge $A_0 = 0$

Kogut-Susskind formalism

Open boundary condition

Gauss law constraint

Jordan-Wigner transformation

Schwinger model as a spin model [Masazumi Honda et al, 2022]

$$H = H_{ZZ} + H_{\pm} + H_{Z}$$

$$H_{ZZ} \sim \sum_{n} \sum_{1\le k<\ell\le n} Z_k Z_\ell \ , \quad H_{\pm} \sim \sum_{n} (X_n X_{n+1} + Y_n Y_{n+1}) \ , \quad H_Z = \sum_{n} Z_n$$

X_n, Y_n, Z_n : Pauli matrices on the n-th site.

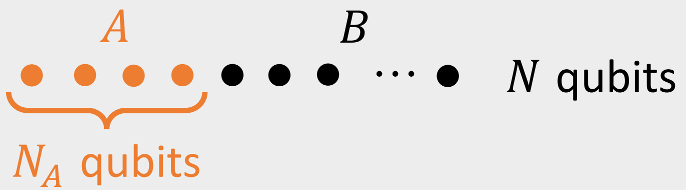
4. Numerical approach using quantum computing

Our setup:

Symmetry

$U(1)$ symmetry, $Q = \frac{1}{2}\sum_n Z_n$

Configuration of subsystem



The procedure of our simulation:

STEP1: Prepare the following initial state.

$$|\phi\rangle = e^{-\mathrm{i}\frac{\phi}{2}\sum_n Y_n} |\, 01 \cdots 01 \,\rangle \qquad , \phi \text{ : Parameter of initial symmetry breaking.}$$

STEP2: Perform the time evolution by second order Trotterization

$$H = H_{ZZ} + H_{XY} + H_Z, \qquad H_{XY} = H_{XY,\mathrm{even}} + H_{XY,\mathrm{odd}}$$

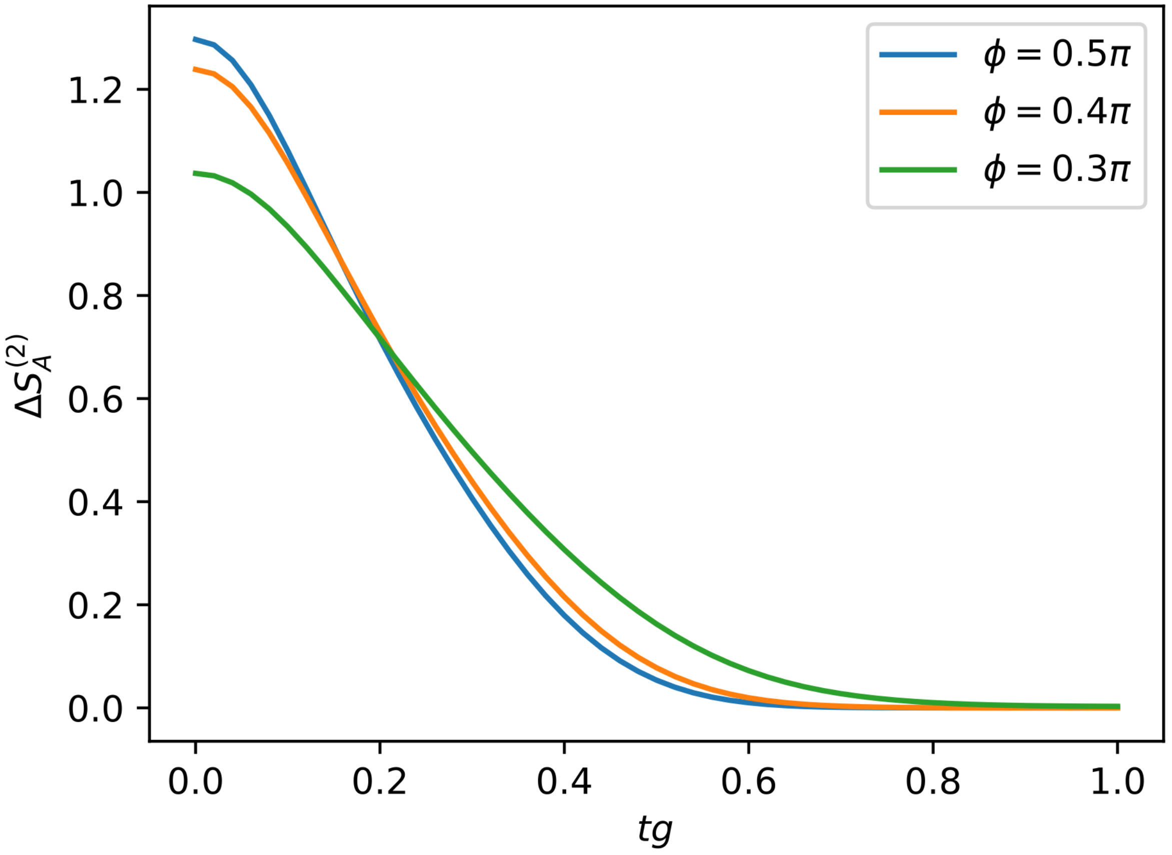
STEP3: Compute $\Delta S_A^{(2)}(t)$ which is obtained by our algorithm.

We performed above quantum simulations using Qulacs, a Python library for classical simulation.

4. Numerical approach using quantum computing

An example of our result:

Degree of symmetry breaking



Real time

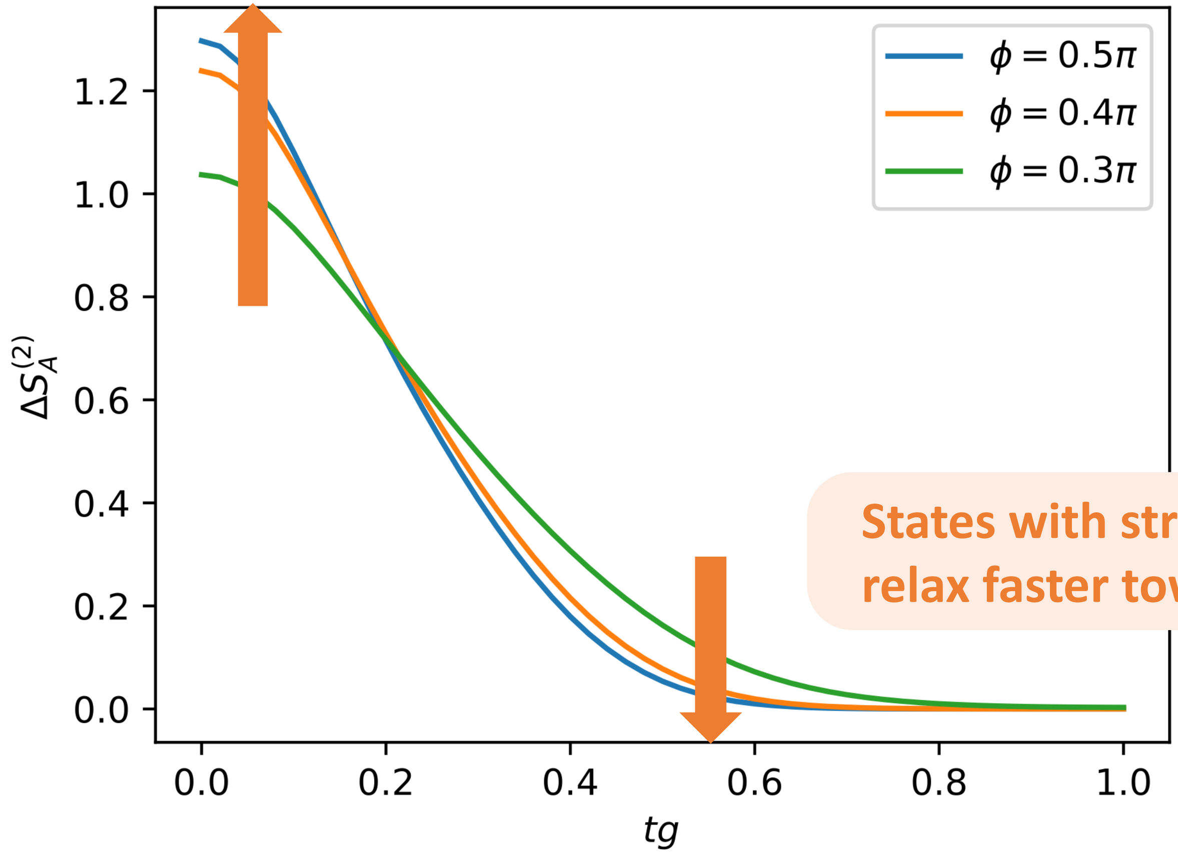
parameters

$N = 18, N_A = 4$
$m/g = 4, \theta = \pi/3$
$ag = 0.25, \delta t = 0.03$
(# of Trotter step) $= 50$

4. Numerical approach using quantum computing

An example of our result:

Degree of symmetry breaking



Real time

parameters

$N = 18, N_A = 4$

$m/g = 4, \theta = \pi/3$

$ag = 0.25, \delta t = 0.03$

(# of Trotter step) $= 50$

States with stronger symmetry breaking relax faster toward symmetry restoration.

Using our quantum algorithm, we demonstrate the quantum Mpemba effect in the Schwinger model.

4. Numerical approach using quantum computing

Resource estimation of our algorithm

There are two errors:

Statistical error of $\langle Z \rangle_{\mathrm{ancilla}}$ in SWAP test

Required number of measurements N_{shot}:

$N_{\mathrm{shot}} = \mathcal{O}(\delta^{-2})$, where δ is standard deviation

Does not depend on system size!

Systematic error of Trotterization

There is an error $\epsilon = \left\| e^{iHt} - U^M(t/M) \right\|$, where $U(t/M)$ is one step of Trotterization.

Required number of gates to achieve an error tolerance ϵ:

(gate complexity) $\leq \mathcal{O}\left(N^{4+1/p} t^{1+1/p} \epsilon^{-1/p}\right)$

[A. M. Childs et al, 2021]

Theoretical upper bound

p: order of Trotterization

Polynomial cost!

Our quantum algorithm is scalable and applicable to large systems.

Outline

1. Introduction
2. Entanglement Asymmetry
3. CFT approach
4. Numerical approach using quantum computing (ongoing)
5. Summary

5. Summary

Summary

- CFT techniques and the properties of symmetry operators enable us to compute entanglement asymmetry.
- We establish the quantum Mpemba effect for non-Abelian symmetries and identify a new type of quantum Mpemba effect.
- In ongoing work, we propose an efficient quantum algorithm for computing entanglement asymmetry.
- We demonstrate the quantum Mpemba effect in the Schwinger model.

Take-home message:

- **CFT is powerful.**
- **The quantum Mpemba effect is one of a promising application of quantum computing.**

5. Summary

Future directions(CFT)

- Other symmetry groups
- Higher Rényi entanglement asymmetries
- Microscopic origin and physical interpretation

Future directions(Quantum computing)

- Feasibility estimates for FTQC and early FTQC (ongoing)
- Implementation on NISQ devices (ongoing)
- Applications to other quantum field theories

Appendix

Entanglement asymmetry for WZW currents

Initial state

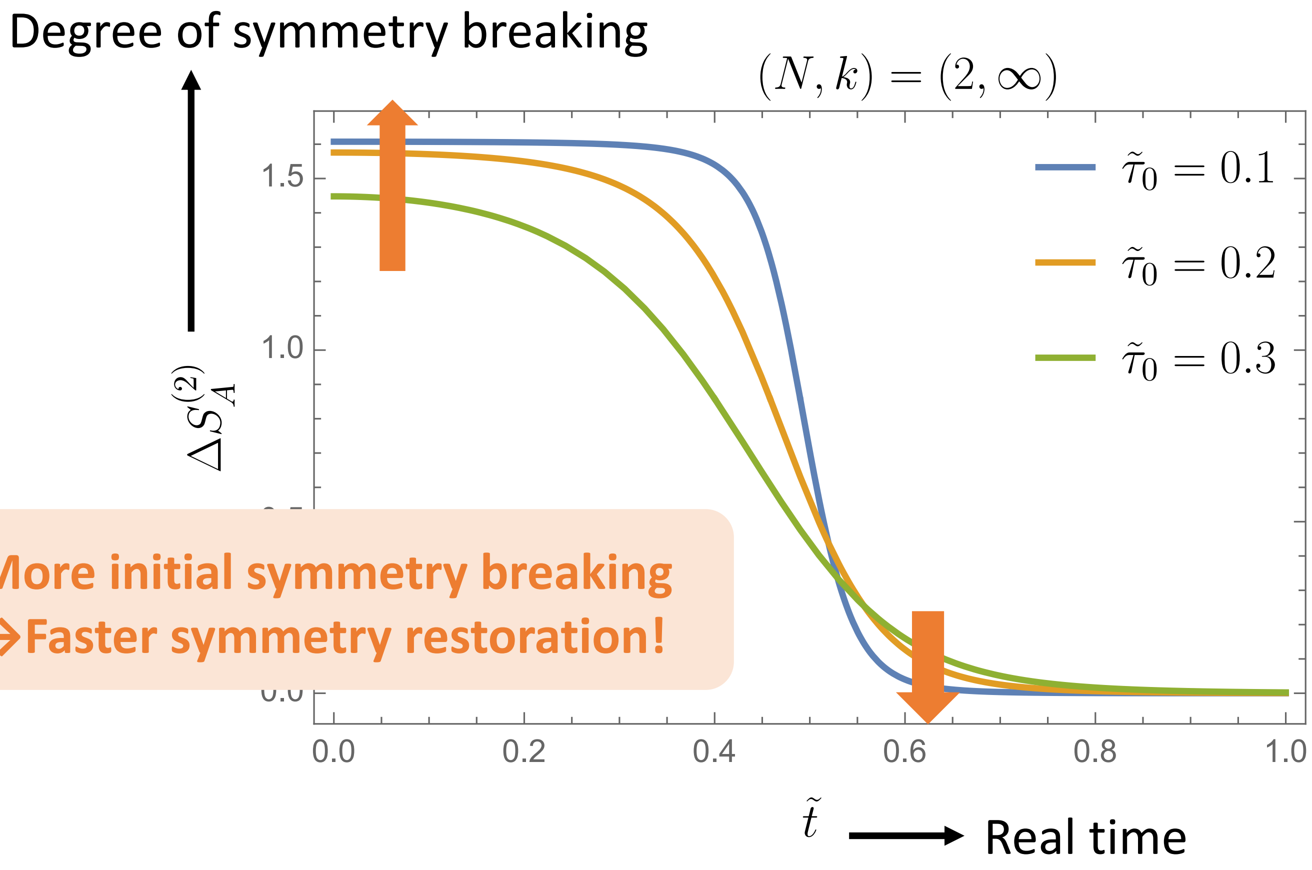
$$|\psi\,(t=0)\rangle = J^a(x,\tau_0)|0\rangle$$

J^a: WZW current in the adjoint rep

$a = 1, \dots, N^2-1$

Change τ_0, while fixing N

(The limit $k \to \infty$ is taken for simplicity)



More initial symmetry breaking
→Faster symmetry restoration!

The quantum Mpemba effect also occurs in this case.

Entanglement asymmetry for WZW currents

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J^a: WZW current in the adjoint rep

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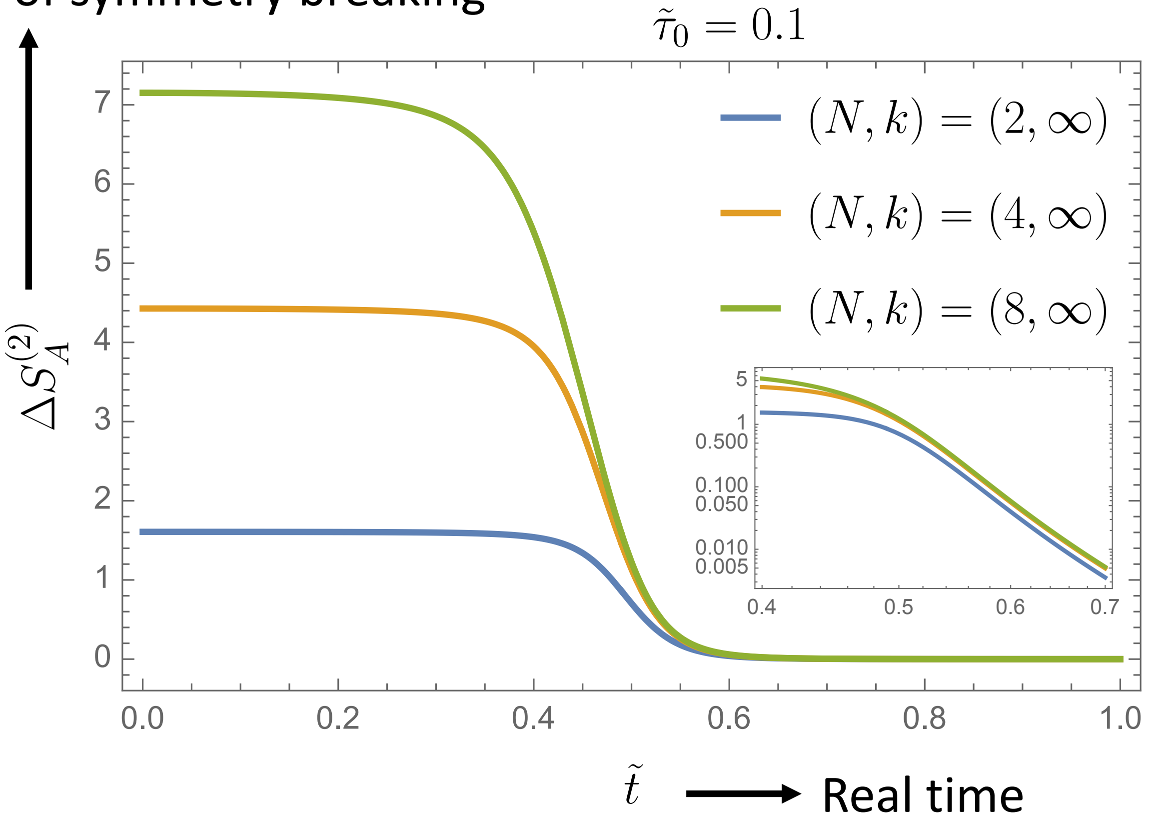
(The limit $k \to \infty$ is taken for simplicity)

Degree of symmetry breaking



Real time

No rank-induced quantum Mpemba effect occurs in this case.

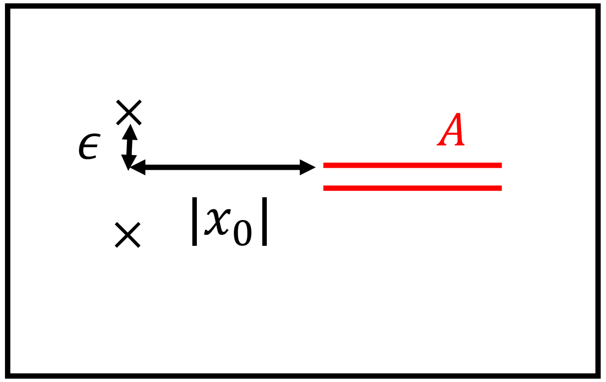
Quasiparticle interpretation

Initial state

$$|\psi_{AB}(t=0)\rangle = \Phi_i(x_0,\tau_0)|0\rangle$$

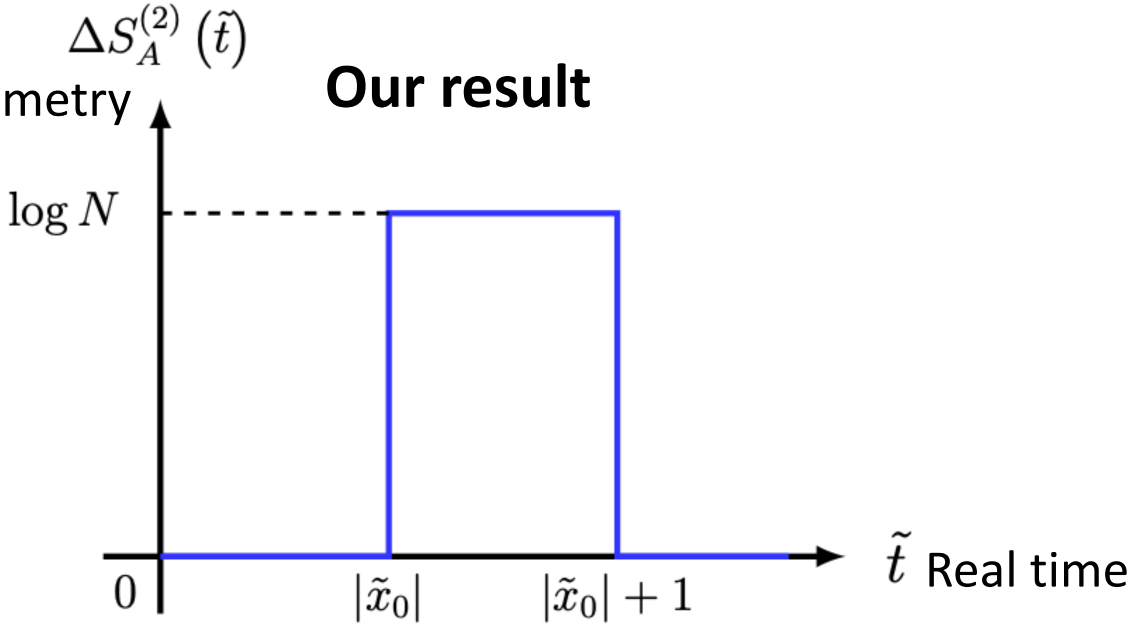
Φ_i: primary field in the fundamental rep

We consider the limit $\tau_0 \to 0$.

$\rho_A =$ 

Degree of symmetry breaking

Our result



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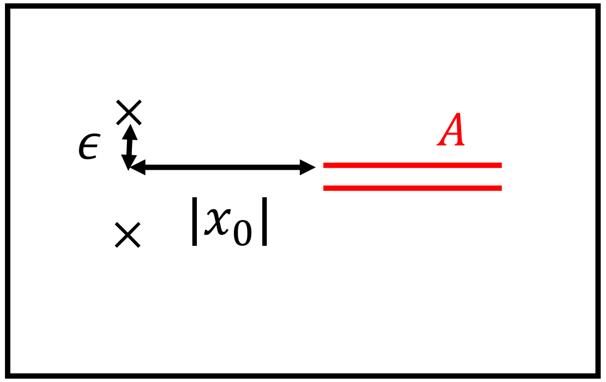
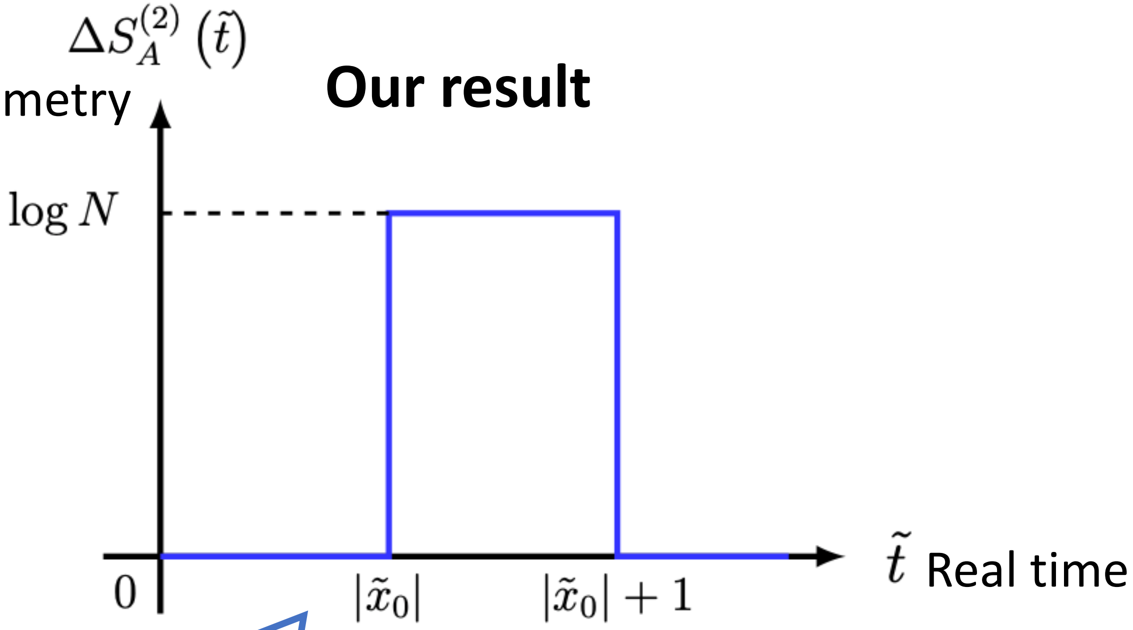
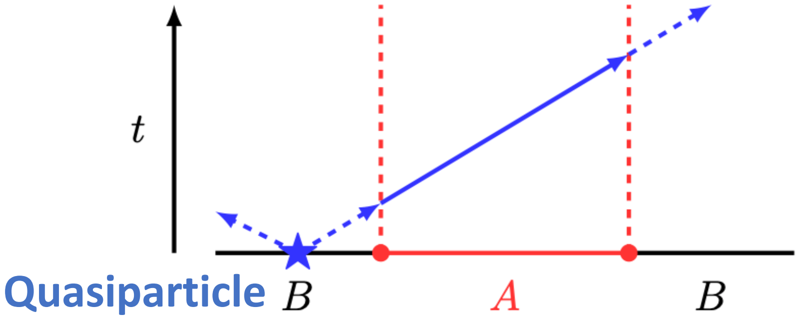
$\rho_A =$ 

Our result

Degree of symmetry breaking



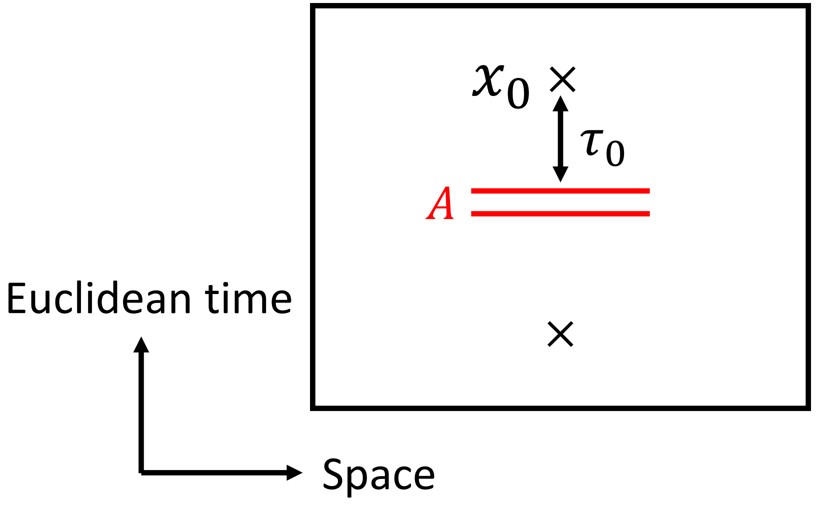
Physical interpretation



Analytical continuation of real time

Let us introduce a complex coordinate $z = x + i\tau$

$$\rho_A(t=0) = \mathrm{Tr}_{\mathrm{A}}\left[\mathcal{O}(z_0, \bar{z}_0)|0\rangle\langle 0|\mathcal{O}(z, \bar{z}_0)^\dagger\right]$$



$$\rho_A(t) = \mathrm{Tr}_{\mathrm{A}}\left[e^{-iHt}\mathcal{O}(z_0, \bar{z}_0)|0\rangle\langle 0|\mathcal{O}(z, \bar{z}_0)^\dagger e^{iHt}\right]$$

$$= \mathrm{Tr}_{\mathrm{A}}\left[\underbrace{e^{-iHt}\mathcal{O}(z_0, \bar{z}_0)e^{iHt}}_{\mathcal{O}(z(t), \bar{z}(t))}|0\rangle\langle 0|e^{-iHt}\mathcal{O}(z, \bar{z}_0)^\dagger e^{iHt}\right]$$

$$\tau = it$$

$$z(t) = x_0 - t + i\tau_0$$

$$\bar{z}(t) = x_0 + t - i\tau_0$$

Detail of our quantum algorithm

To simplify the analysis, we focus on the Rényi entanglement asymmetry:

$$\Delta S_A^{(n)} \equiv \frac{1}{n-1}\left(\log \mathrm{Tr}_A[\rho_A^n] - \log \mathrm{Tr}_A\left[\rho_{A,S}^n\right]\right), \quad \lim_{n\to 1} \Delta S_A^{(n)} = \Delta S_A$$

For concreteness, we consider $n = 2$.

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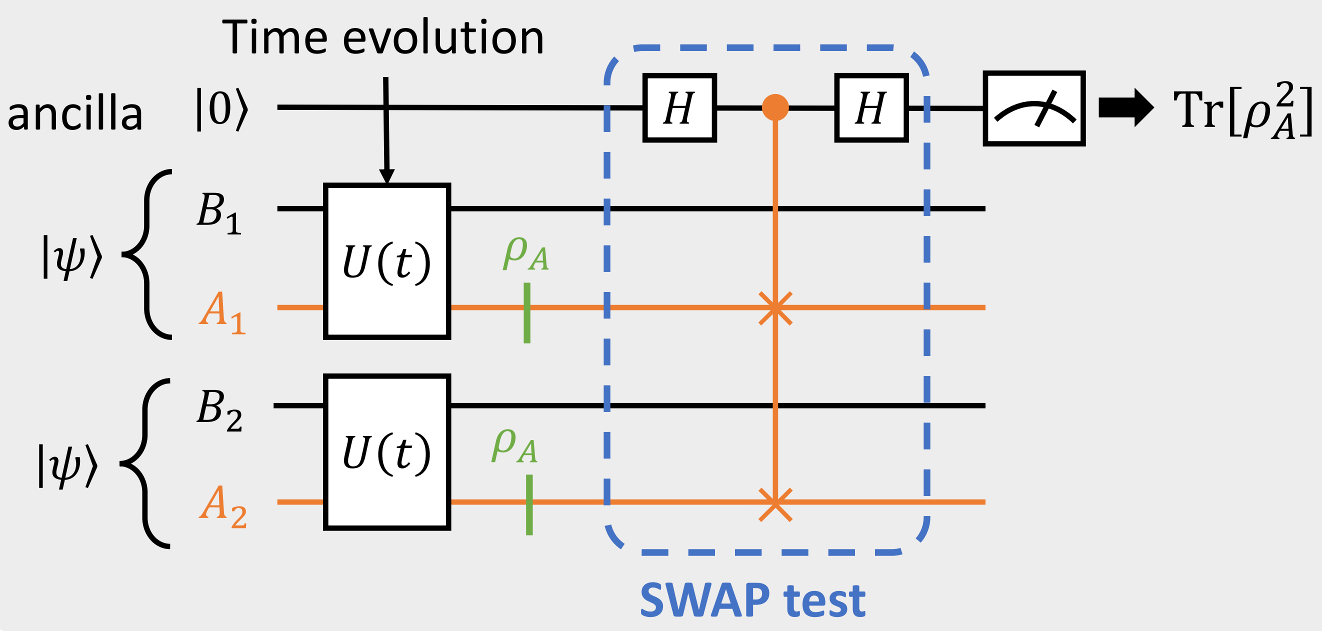
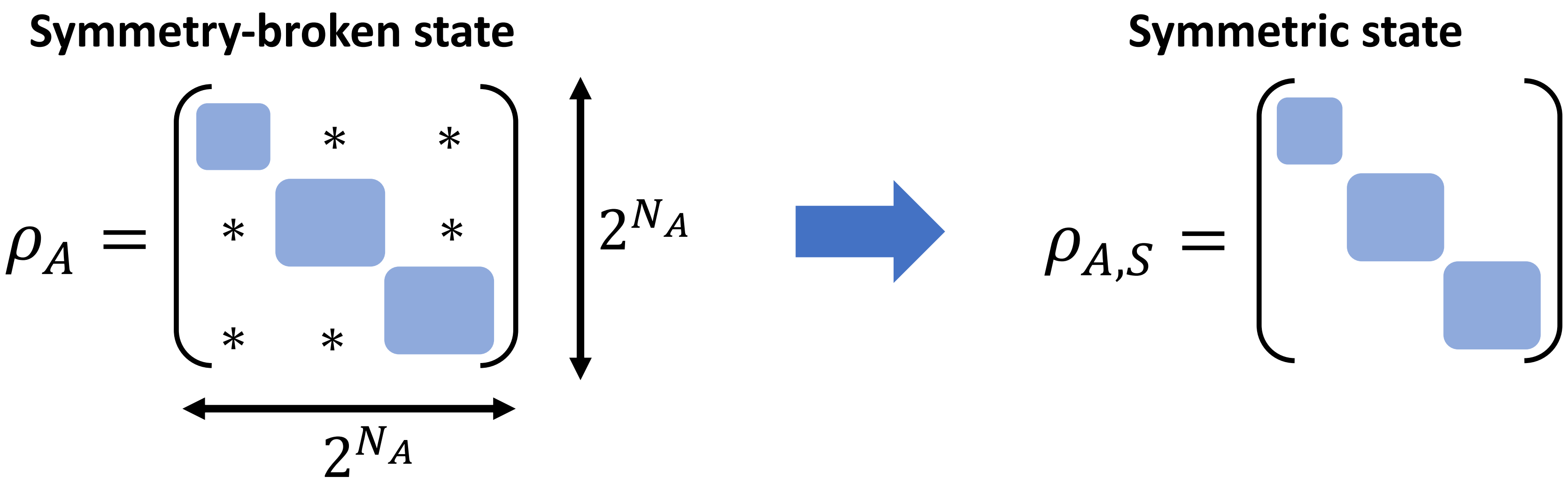
Quantum circuit for computing $\mathrm{Tr}_A[\rho_A^2]$



How can we compute $\mathrm{Tr}_A[\rho_{A,S}^2]$?

(Our work)

Detail of our quantum algorithm

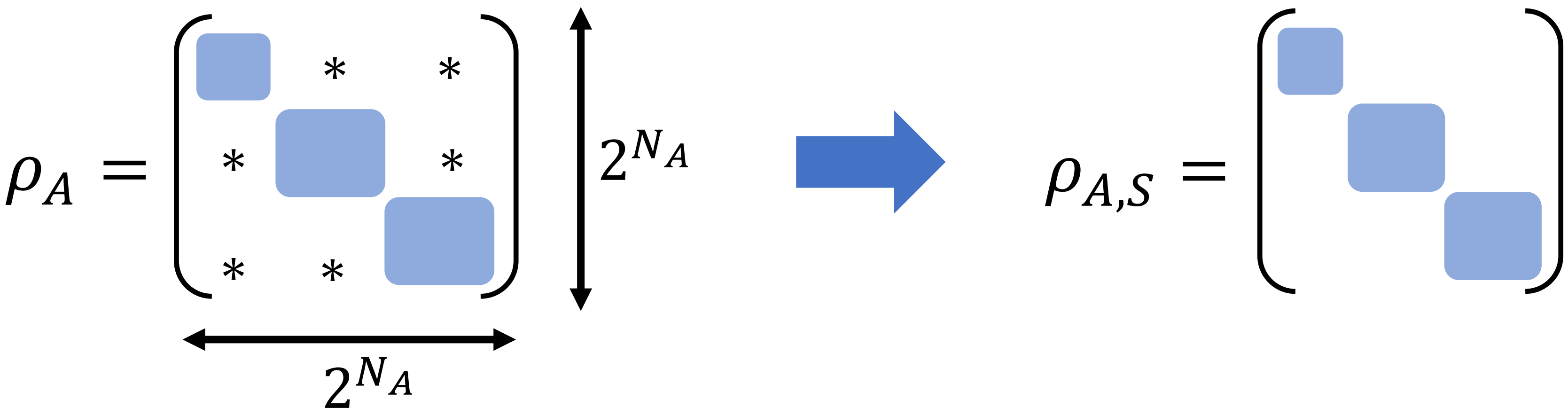


Naively, one need to remove $\mathcal{O}(2^{2N_A})$ off-diagonal elements.

Question: How can we realize this efficiently in a quantum circuit?

Detail of our quantum algorithm

Symmetry-broken state **Symmetric state**



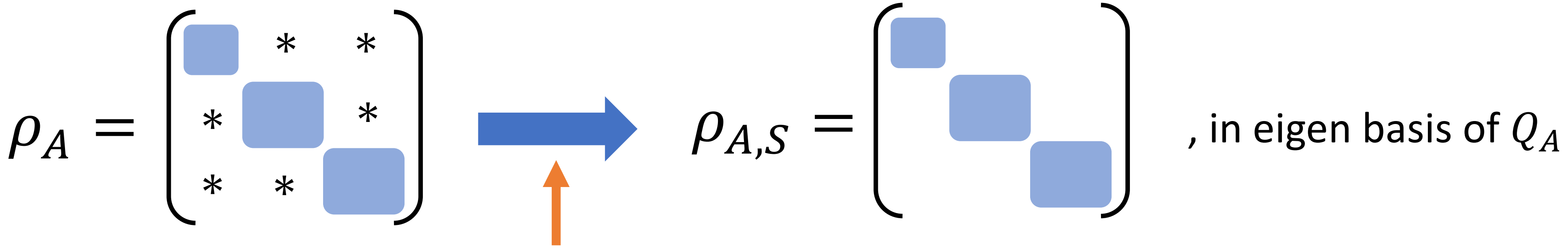
Naively, one need to remove $\mathcal{O}(2^{2N_A})$ off-diagonal elements.

Question: How can we realize this efficiently in a quantum circuit?

Our idea: Reformulating it as “decoherence”

Detail of our quantum algorithm

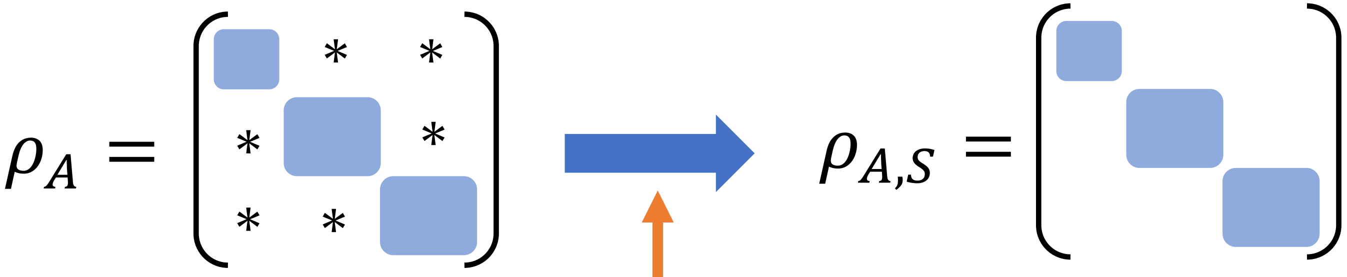
Decoherence: the loss of quantum coherence due to interactions with environment.



Treat this as a decoherence.

Detail of our quantum algorithm

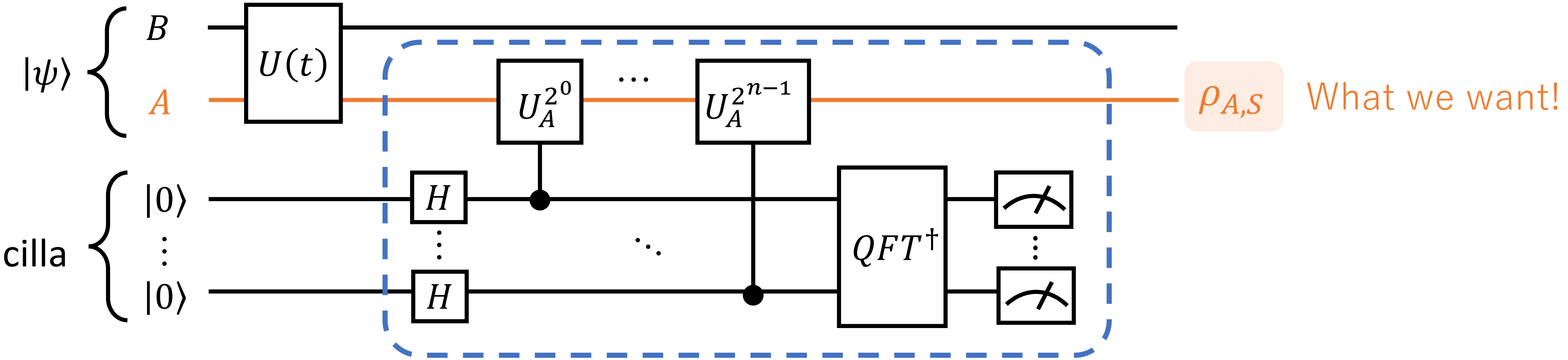
Decoherence: the loss of quantum coherence due to interactions with environment.



, in eigen basis of Q_A

Treat this as a decoherence.

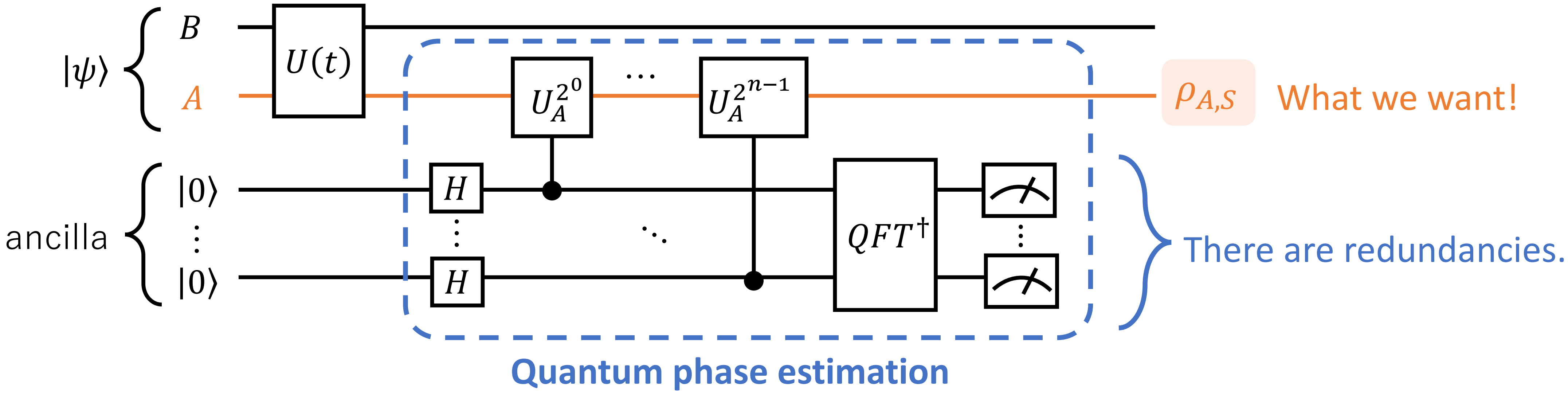
We implement this decoherence by employing a **quantum phase estimation** of $U_A = e^{2\pi i Q_A/2^n}$, $n > \log_2 N_A$



Quantum phase estimation

$\rho_{A,S}$ **can be prepared in a quantum circuit!**

Detail of our quantum algorithm



simplify

We can drastically simplify the quantum circuit.

Technical detail

$$\rho_A = \begin{pmatrix} \blacksquare & * & * \\ * & \blacksquare & * \\ * & * & \blacksquare \end{pmatrix} \qquad \rho_{A,S} = \begin{pmatrix} \blacksquare & & \\ & \blacksquare & \\ & & \blacksquare \end{pmatrix}$$

, in eigenbasis of Q_A

$$\mathrm{Tr}[\rho_A \rho_{A,S}] = \mathrm{Tr} \begin{pmatrix} \blacksquare & & \\ & \blacksquare & \\ & & \blacksquare \end{pmatrix} \begin{pmatrix} \blacksquare & * & * \\ * & \blacksquare & * \\ * & * & \blacksquare \end{pmatrix} = \mathrm{Tr} \begin{pmatrix} \blacksquare^2 & & \\ & \blacksquare^2 & \\ & & \blacksquare^2 \end{pmatrix} = \mathrm{Tr}[\rho_{A,S}^2]$$

In general, the following identity holds

$$\mathrm{Tr}[\rho_A \rho_{A,S}^{n-1}] = \mathrm{Tr}[\rho_{A,S}^n]$$