Entanglement Asymmetry and Quantum Mpemba Effect for Non-Abelian Global Symmetry

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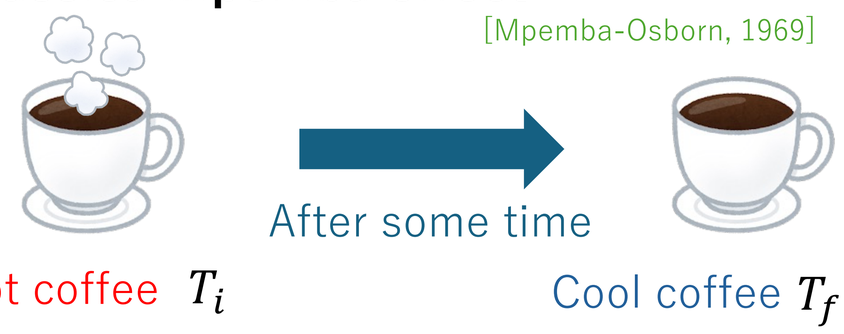
1. Introduction

Classica Mpemba effect

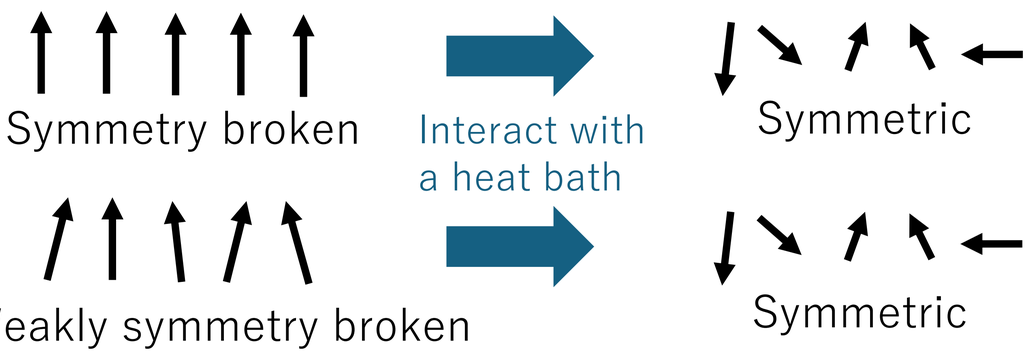
[Mpemba-Osborn, 1969]



Hot coffee cools faster than warm coffee under certain conditions. ➡ Anomalous relaxation!

Quantum Mpemba effect (symmetry)

Let's consider spin flip symmetry.



The more symmetry is initially broken, the faster it is restored. ➡ Anomalous symmetry restoration!

Previous studies

Quantum Mpemba effect has been studied in:

- Quantum spin chains (1d, 2d, integrable system)
- Experimental observation in quantum computer
- (1+1)d CFT with $U(1)$ symmetry.

Previous studies focus on Abelian symmetry such as $U(1)$ or $\mathbb{Z}_n$.

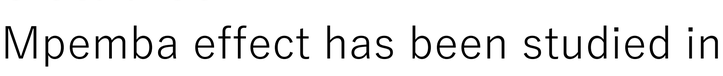
➡ Quantum Mpemba effect for non-Abelian symmetries remains unexplored.

Our work

- We showed **quantum Mpemba effect for non-Abelian symmetries** by employing CFT.
- Moreover, we uncover **a new type of quantum Mpemba effect.**

2. Setting

Quench by symmetric Hamiltonian



$|\psi_{AB}(0)\rangle$: symmetry broken initial state

$|\psi_{AB}(t)\rangle = e^{-iHt}|\psi_{AB}(0)\rangle$, H : symmetric Hamiltonian

➡ The symmetry is restored on subsystem A.

Rényi entanglement asymmetry

[Ares et al, 2022]

(REA) $\Delta S_A^{(n)} \equiv \frac{1}{1-n}\left(\log \mathrm{Tr}\left[\rho_{A,S}^n\right] - \log \mathrm{Tr}[\rho_A^n]\right)$

ρ_A : density matrix, $\rho_{A,S}$: symmetrized density matrix

REA is a quantitative measure of symmetry breaking.

3. Analytical approach

We consider $\widehat{su}(N)_k$ Wess-Zumino-Witten (WZW) model.

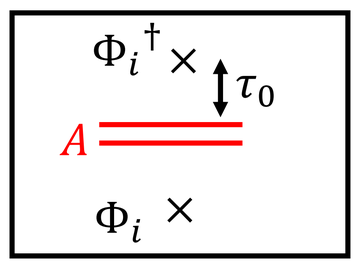
Initial state

$|\psi_{AB}(0)\rangle = \Phi_i(x_0, \tau_0)|0\rangle$

Φ_i : primary field $(i = 1, \ldots, N)$

τ_0 : parameter of symmetry breaking

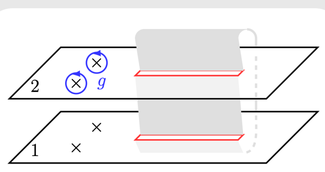
$\rho_A =$



Euclidean path integral

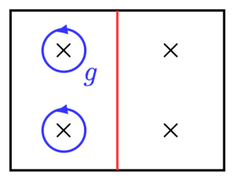
Replica trick

[Benini et al, 2024]



Conf. transf.

[He et al, 2024]



4pt function

[Knizhnik et al, 1984]

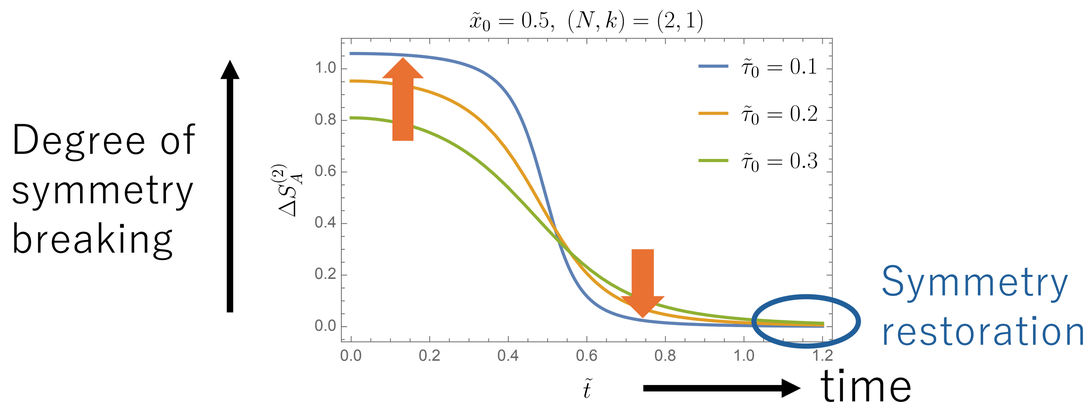
$\langle \Phi_i \Phi_j{}^\dagger \Phi_k \Phi_\ell{}^\dagger \rangle$

➡ Solvable

4. Results

Quantum Mpemba effect for SU(N)

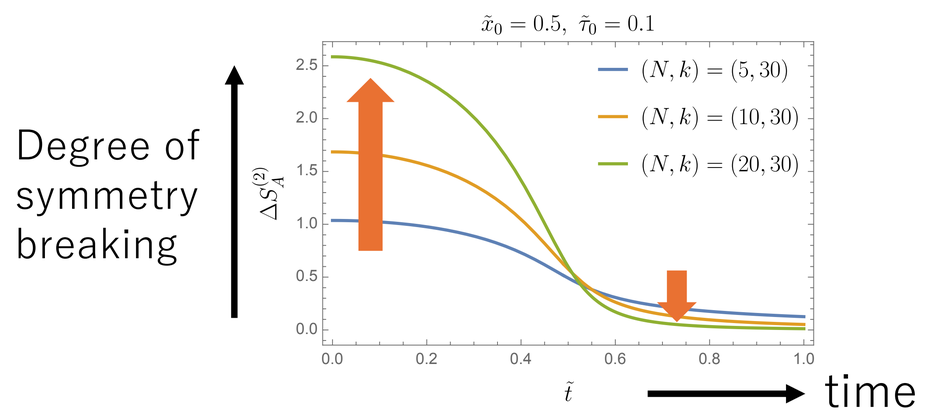
$SU(N)$ symmetry restoration for various τ_0



States with stronger symmetry breaking relax faster toward symmetry restoration.

New type of quantum Mpemba effect

Rank N dependence with fixed τ_0



Increasing the rank leads to stronger initial symmetry breaking but faster symmetry restoration.

➡ **New finding!**

5. Summary

- We show the existence of quantum Mpemba effect for non-Abelian symmetry using WZW model.
- Furthermore, we uncover a new type of quantum Mpemba effect emerging between different ranks.

Other topics (not shown here):
quasi particle picture, level k dependence, another initial state

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