A Scalable Quantum Algorithm for the Quantum Mpemba Effect

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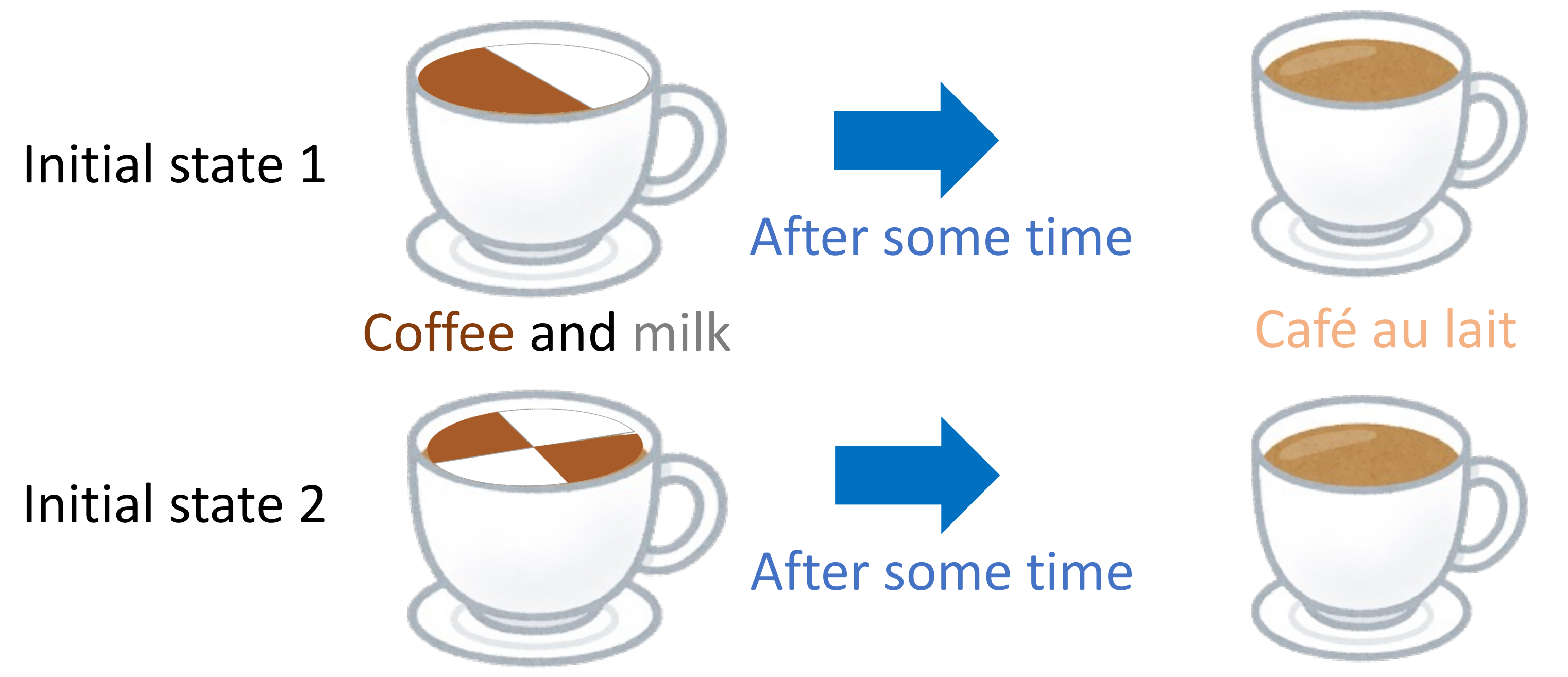
1. Introduction

Quantum Mpemba Effect

[Mpemba-Osborn, 1969] [Ares et al, 2022]

Symmetry broken state **Symmetric state**



Intuitively, one expects the initial state 2 relax faster than the initial state 1.
However, the opposite behavior is observed under certain conditions!

Quantum Mpemba Effect (QME)

The more symmetry is initially broken, the faster it is restored.

An anomalous out-of-equilibrium relaxation.

Background

The QME has attracted increasing attention across diverse fields, including condensed matter physics, high-energy physics, and quantum information science.

Challenges

- Analytical studies are limited to integrable models and conformal field theories.
- The origin of the QME remains elusive.
- Classical simulations suffer from exponential scaling with system size.
- Quantum field theories remain largely inaccessible.

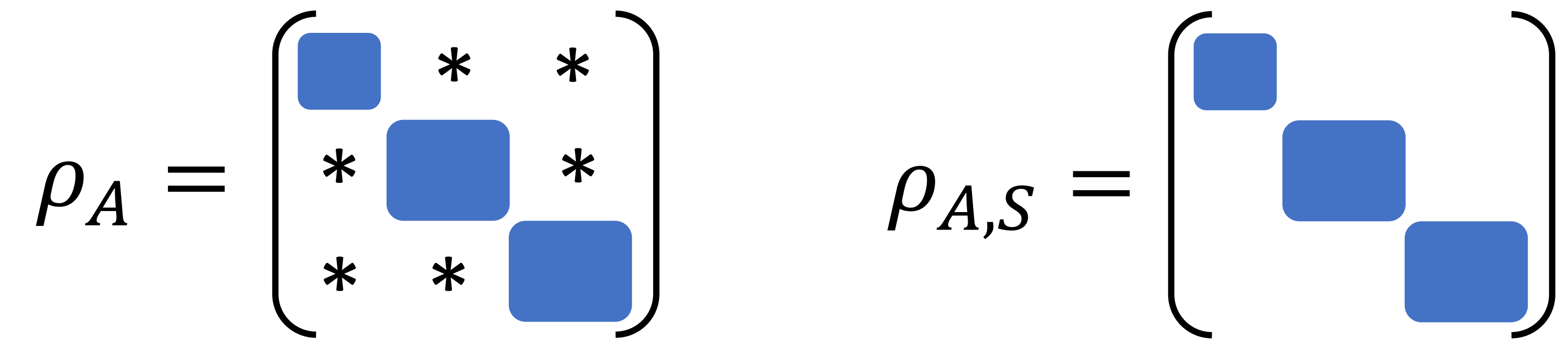
A scalable approach is needed to investigate the QME in large quantum system!

Our work

- We develop a quantum algorithm for the QME.
- Our approach avoids exponential computational cost and is therefore scalable to large systems.
- Application to the Schwinger model (1+1d quantum electrodynamics)
- We demonstrate that our algorithm enables the investigation of the QME in a quantum field theory.

2. Entanglement Asymmetry

Let A be a subsystem of interest and consider the reduced density matrix in the eigenbasis of the symmetry charge.



Off-diagonal = symmetry breaking Diagonal = symmetric state

Rényi Entanglement Asymmetry (REA)

$$\Delta S_A^{(n)} \equiv \frac{1}{1-n}\left(\log \mathrm{Tr}\left[\rho_{A,S}^n\right] - \log \mathrm{Tr}[\rho_A^n]\right)$$

○ REA provide a quantitative measure of symmetry breaking and serves as a probe of the QME

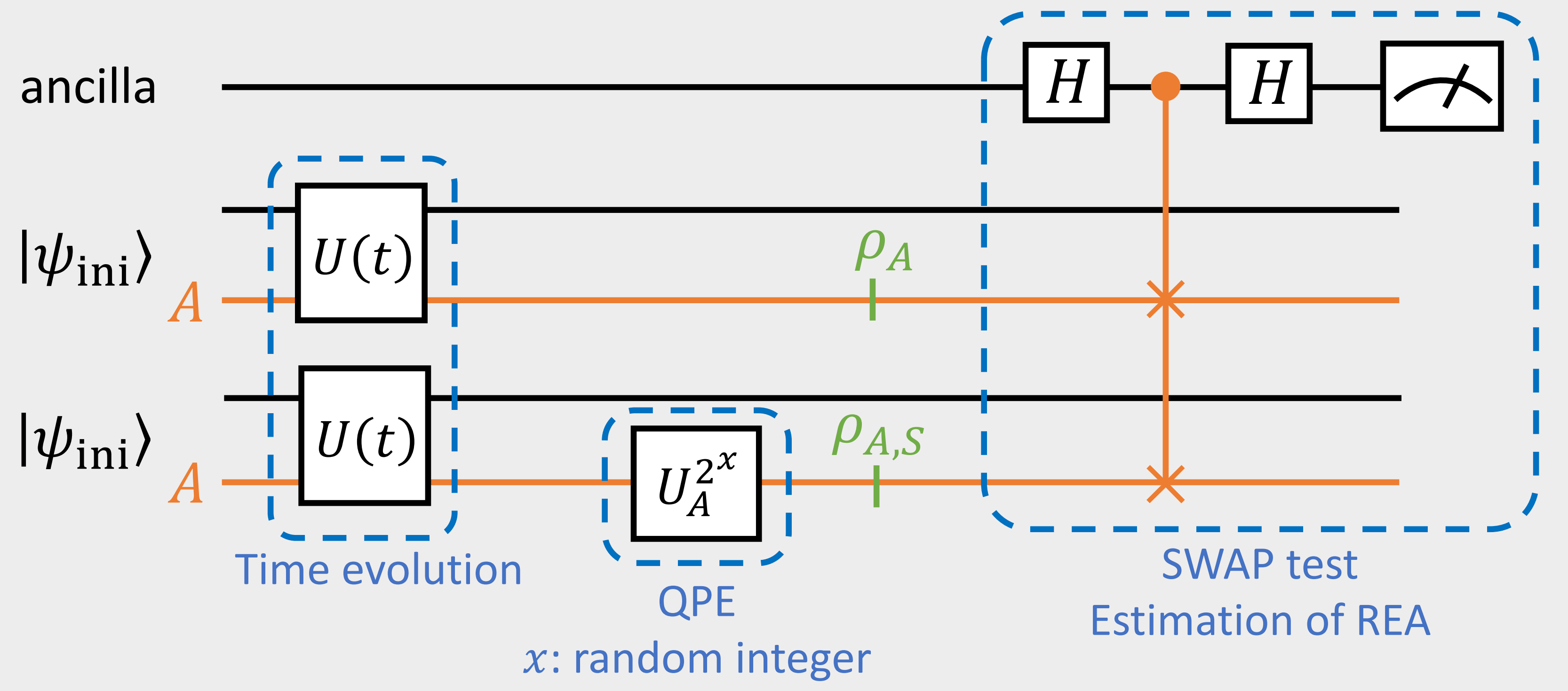
▲ Classical simulation is exponentially expensive:
$\mathcal{O}(2^{3N_A})$, where N_A is the system size

3. Quantum Algorithm

Our idea

- **Quantum phase estimation(QPE)** → Symmetry projection
- **SWAP test** → REA estimation
- Combining QPE and SWAP test, we can efficiently investigate the QME.

Our quantum circuit $(n = 2)$



Key Features

- Measurement cost is independent of the system size.
  $N_{\mathrm{shot}} = \mathcal{O}(\delta^{-2})$, where δ is statistical error of SWAP test.
- Gate complexity scales polynomially with the system size.
  (Gate complexity)$\leq \mathcal{O}(N^{\#})$, where N is system size

No exponential computational cost!

Toward scalable investigations of the QME in quantum field theories!

4. Application

Schwinger model (1+1d quantum electrodynamics)

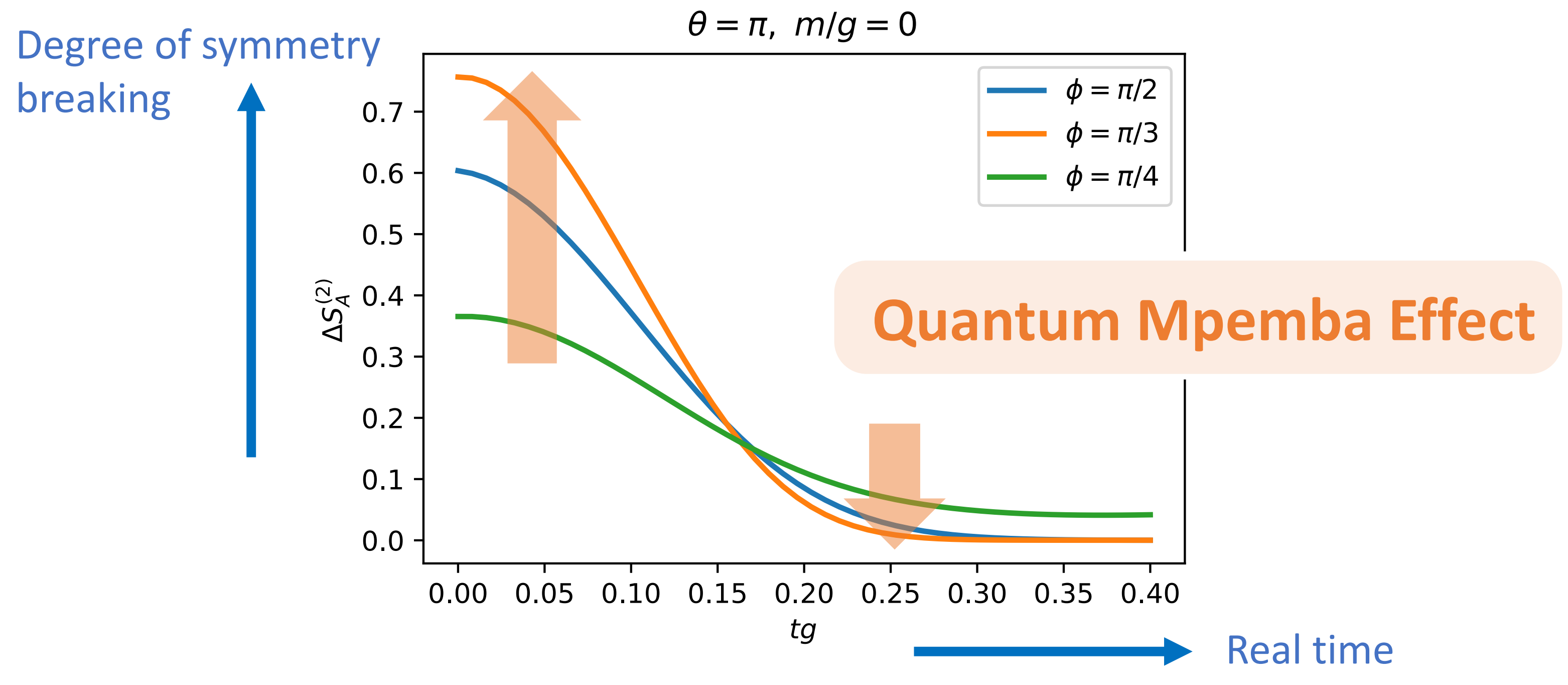
After lattice discretization, this model is described by the following Hamiltonian.

$$H = H_{ZZ} + H_{\pm} + H_Z$$

$$H_{ZZ} \sim \sum_n \sum_{1\leq k<\ell\leq n} Z_k Z_\ell \ , \quad H_{\pm} \sim \sum_n (X_n X_{n+1} + Y_n Y_{n+1}) \ , \quad H_Z \sim \sum_n Z_n$$

X_n, Y_n, Z_n: Pauli operators.

Initial state: $|\psi_{\mathrm{ini}}\rangle = e^{i\frac{\phi}{2}\sum_n Y_n}|00\cdots 0\rangle$,
where ϕ is the parameter of initial symmetry breaking.



Our results demonstrate that quantum computers can be used to investigate the QME in quantum field theories!

5. Summary

- We propose a scalable quantum algorithm for the QME, where measurement cost is independent of system size, and gate complexity scales polynomially.
- Using the Schwinger model as an example, we demonstrate the feasibility of investigating the QME in quantum field theories.

Future directions

- Implementation on NISQ devices (ongoing).
- Hybrid quantum-classical approaches for state preparation.

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