Quantum simulation of quantum Mpemba effect in Schwinger model

Harunobu Fujimura

The University of Osaka

大阪大学
THE UNIVERSITY OF OSAKA

2026/1/27 15:00~15:20

Based on ongoing work with
Keisuke Fujii, Masazumi Honda and Duc Truyen Le.

1. Introduction

The main topic of this talk:

Quantum computing → apply → **Symmetry breaking and its restoration in QFT**

Take home message :

Quantum computing enables us to study anomalous symmetry restoration dynamics in quantum field theory.

1. Introduction

The main topic of this talk:

Quantum computing → apply → **Symmetry breaking and its restoration in QFT**

We study a counterintuitive phenomenon known as the "quantum Mpemba effect".

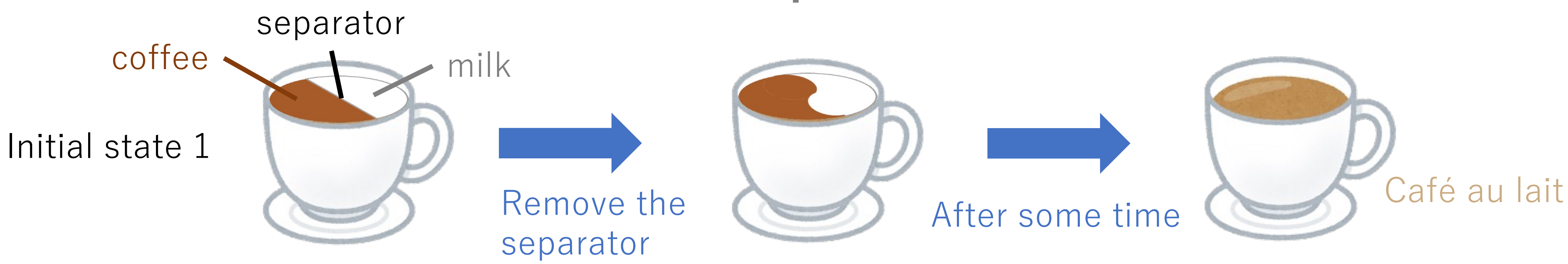
Take home message :

Quantum computing enables us to study anomalous symmetry restoration dynamics in quantum field theory.

1. Introduction

Mpemba effect is an anomalous out-of-equilibrium relaxation.
[E B Mpemba and D G Osborn, 1969]

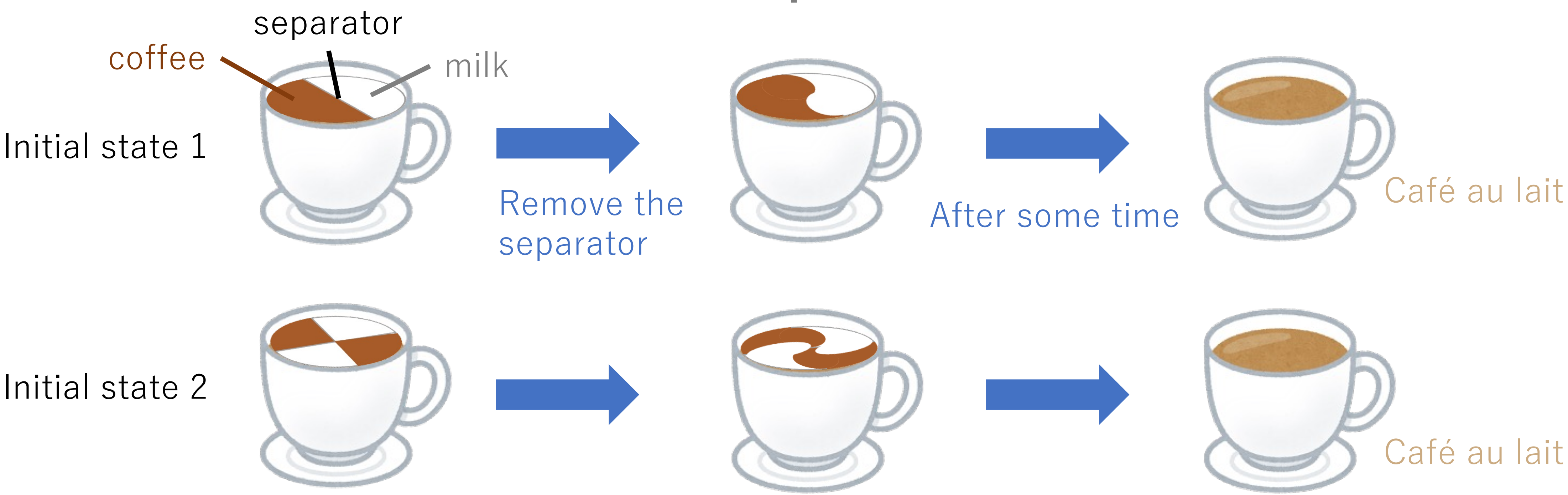
Metaphor



1. Introduction

Mpemba effect is an anomalous out-of-equilibrium relaxation.
[E B Mpemba and D G Osborn, 1969]

Metaphor



Intuitively, one expects the initial state 2 to relax faster than the initial state 1.

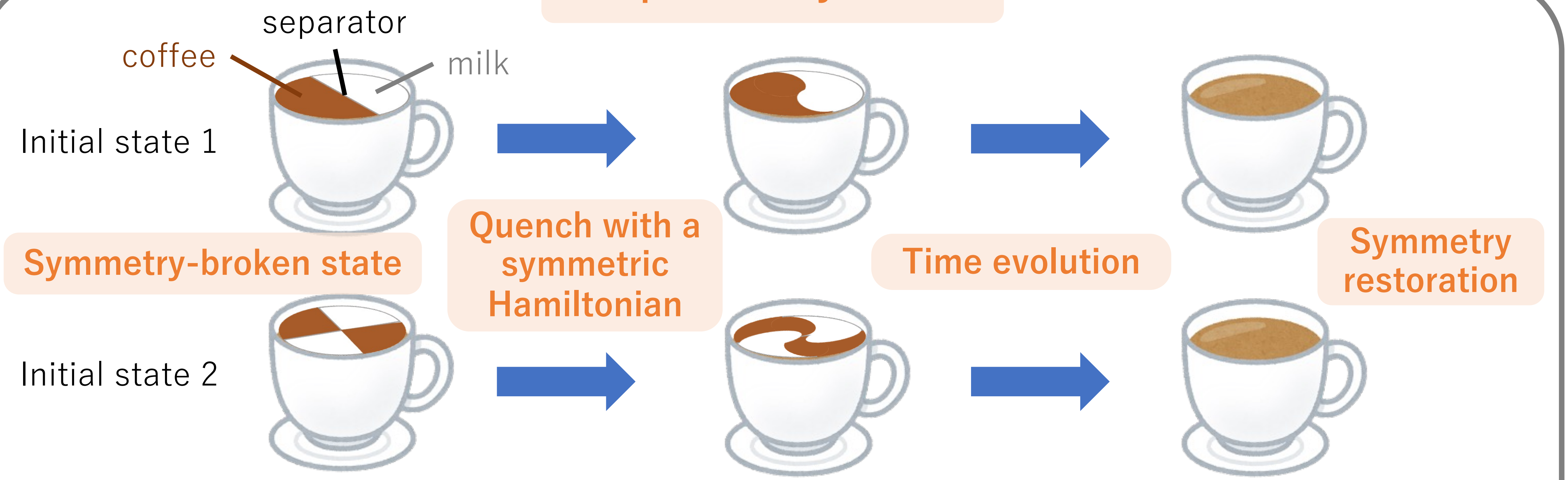
However, under certain conditions, the opposite behavior is observed.

➡ **Counterintuitive phenomenon!**

1. Introduction

Mpemba effect is an anomalous out-of-equilibrium relaxation.
[E B Mpemba and D G Osborn, 1969]

In quantum system…

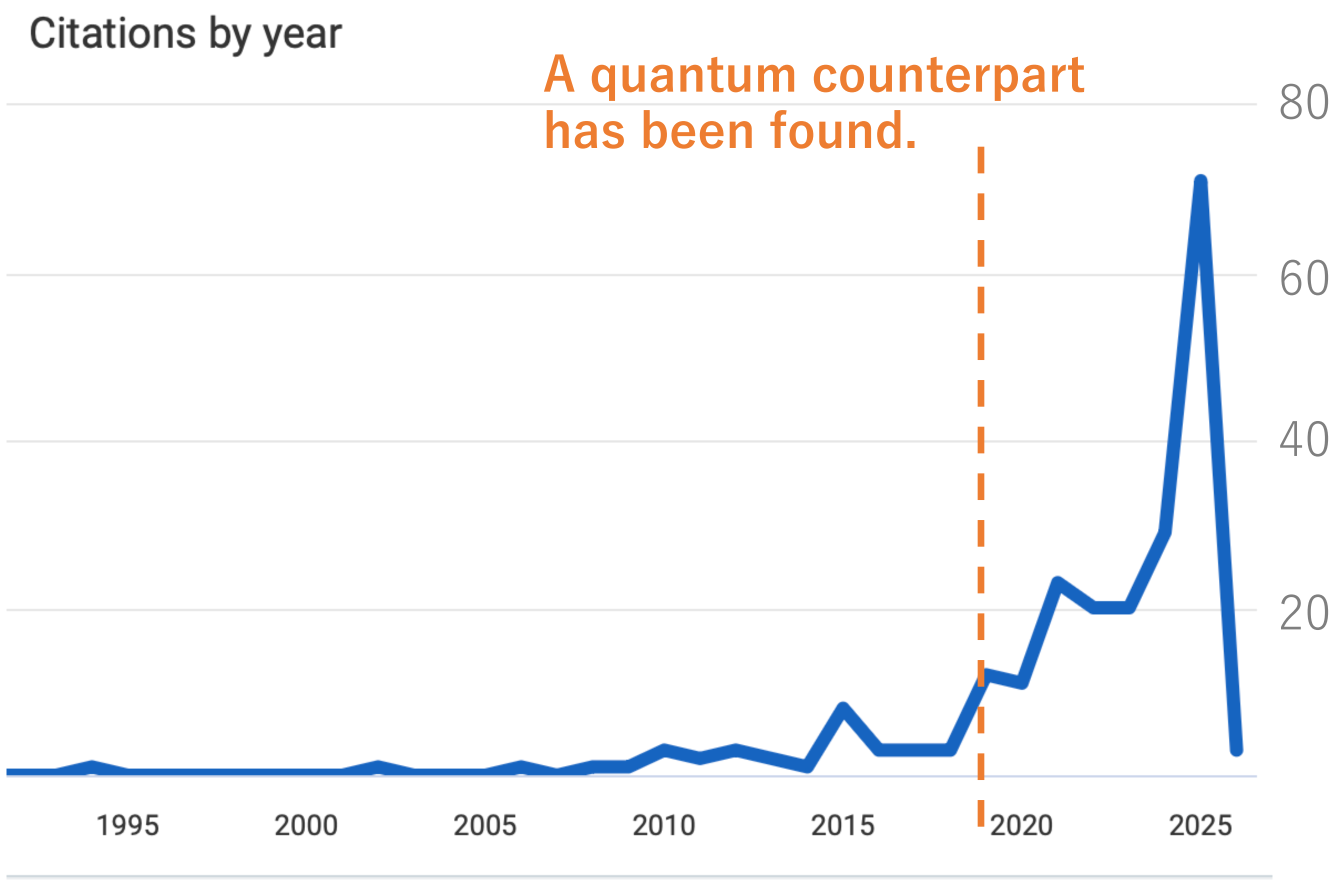


Quantum Mpemba effect

The more symmetry is initially broken, the faster it is restored.

1. Introduction

The Mpemba effect was first observed in classical systems, but…
[E B Mpemba and D G Osborn, 1969]



The quantum Mpemba effect is a rapidly developing topic!

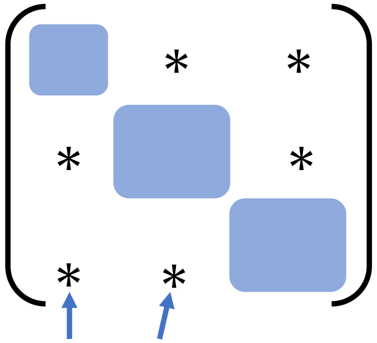
1. Introduction

We assume that the theory has a symmetry with charge

$$Q = Q_A + Q_B$$

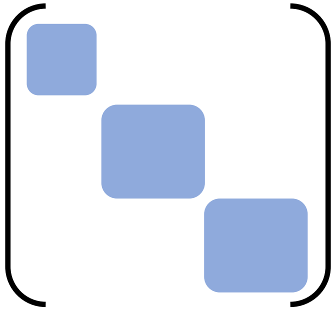
A = Subsystem of interest, B = Environment.

Symmetry-broken state

$\rho_A =$ 

Off-diagonal elements

Symmetric state

$\rho_{A,S} =$ 

, in eigenbasis of Q_A

Block diagonal

Entanglement Asymmetry : A quantifier of symmetry breaking

[Ares-Murciano-Calabrese, 2022]

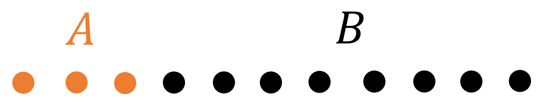
$$\Delta S_A \equiv \Delta S(\rho_A | \rho_{A,S}) = \mathrm{Tr}_A[\rho_A(\log \rho_A - \log \rho_{A,S})]$$

Relative entropy

1. Introduction

Typical protocol to investigate the quantum Mpemba effect.

Quench dynamics with a symmetric Hamiltonian



Spin chain or QFT

Total system$=A \cup B$

A : subsystem of interest

STEP1: Prepare an initial state $|\psi_{AB}(0)\rangle$ that **explicitly** breaks the symmetry.

STEP2: Perform the time evolution.

$$|\psi_{AB}(t)\rangle = e^{-iHt}|\psi_{AB}(0)\rangle,$$ where H is a symmetric Hamiltonian.

STEP3: Compute the entanglement asymmetry at time t.

$$\rho_A(t), \rho_{A,S} \Rightarrow \Delta S_A(t)$$

The entanglement asymmetry captures the real-time dynamics of symmetry restoration.

1. Introduction

Previous studies of quantum Mpemba effect

- Condensed matter physics (integrable and non-integrable spin chains)
- High-energy physics (conformal field theories)
- Quantum information theory (random circuit, resource theory dynamics)

Challenges

- Computing the entanglement asymmetry beyond CFTs or integrable systems.
- Numerical approaches by classical computer : $\mathcal{O}(N_A^2)$ → **not scalable!** N_A : the size of subsystem
- Monte Carlo methods are not applicable due to the sigh problem.

Large systems, including quantum field theories, are therefore difficult to access.

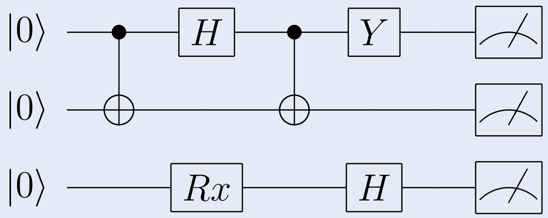
1. Introduction

Challenges

- Numerical approach → **not scalable** ✖
- Monte Carlo methods ✖
- Quantum field theories ▲

Our approach ←

Quantum computing



Our work

- We propose a **scalable** quantum algorithm that efficiently compute the entanglement asymmetry.
- As an application, we study Schwinger and demonstrate that our quantum algorithm can be used to investigate the quantum Mpemba effect in quantum field theories.

Outline

1. Introduction

2. Quantum Algorithm for Entanglement Asymmetry

3. Quantum Mpemba Effect in Schwinger Model

4. Summary

Outline

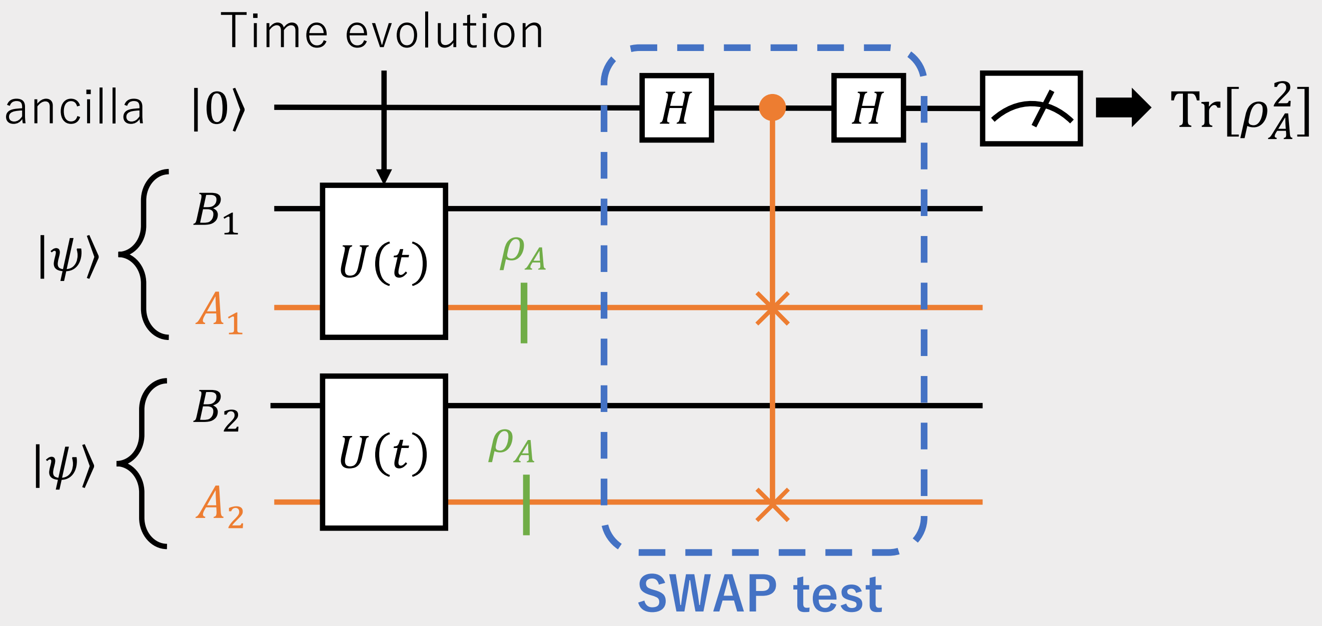
2. Quantum Algorithm for Entanglement Asymmetry

To simplify the analysis, we focus on the Rényi entanglement asymmetry:

$$\Delta S_A^{(n)} \equiv \frac{1}{n-1}\left(\log \mathrm{Tr}_A[\rho_A^n] - \log \mathrm{Tr}_A[\rho_{A,S}^n] \right), \quad \lim_{n\to 1} \Delta S_A^{(n)} = \Delta S_A$$

For concreteness, we consider $n = 2$.

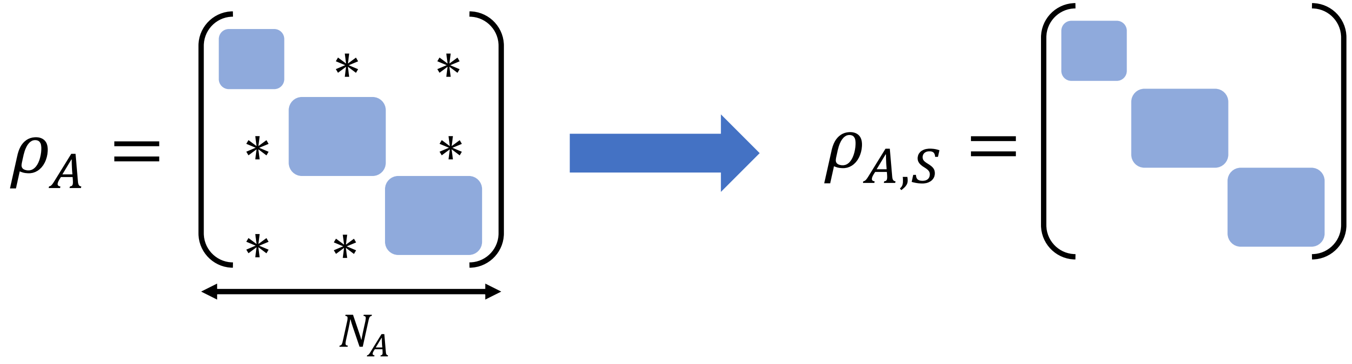
Quantum circuit for computing $Tr_A[\rho_A^2]$



How to realize $\rho_{A,S}$ in a quantum circuit?

2. Quantum Algorithm for Entanglement Asymmetry

Naively, one need to remove $\mathcal{O}(N_A^2)$ off-diagonal elements.

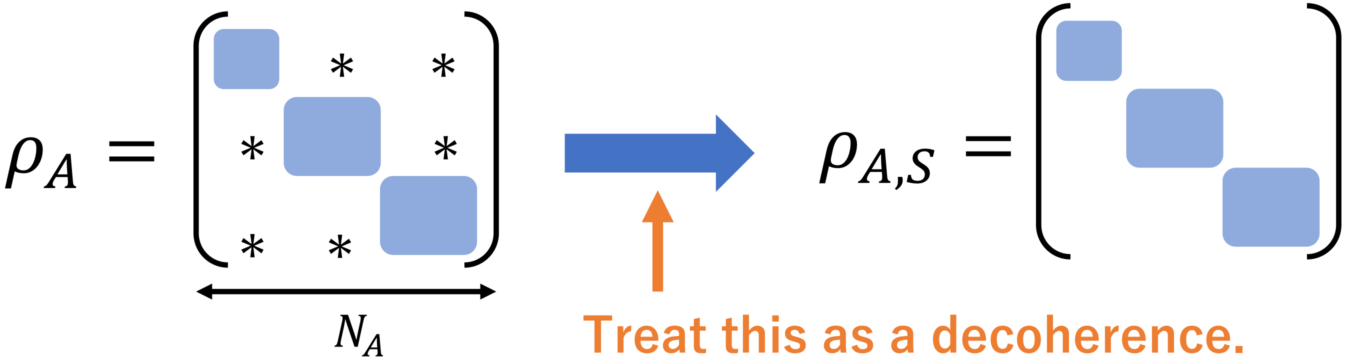


, in eigenbasis of Q_A

How can this be done efficiently?

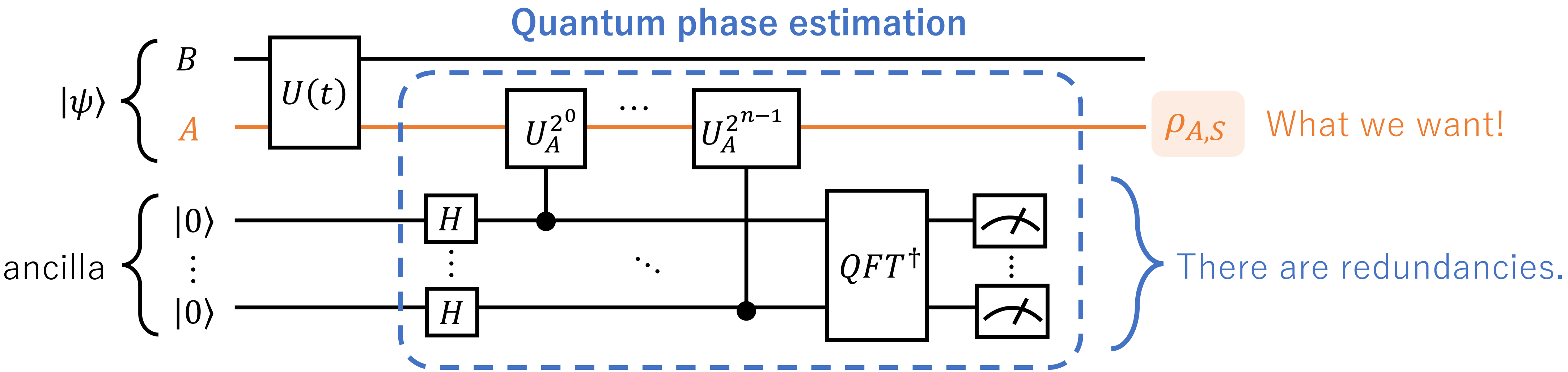
2. Quantum Algorithm for Entanglement Asymmetry

Naively, one need to remove $\mathcal{O}(N_A^2)$ off-diagonal elements.



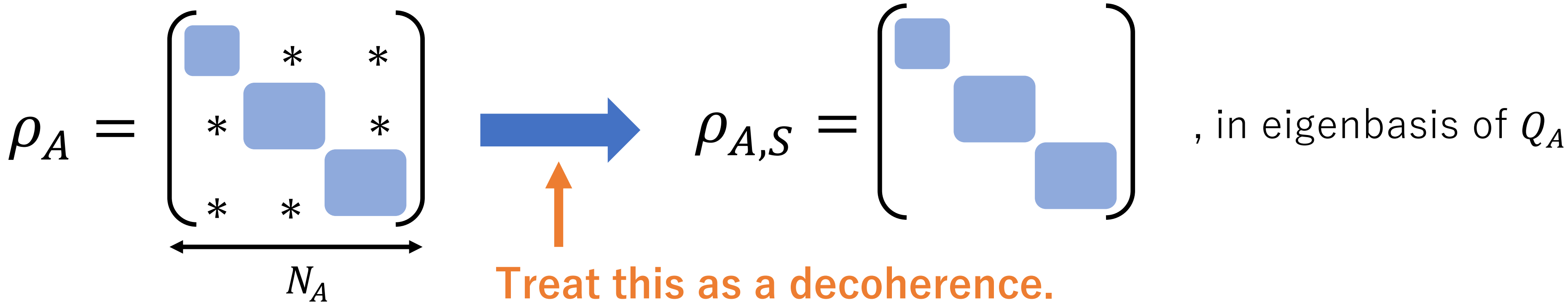
, in eigenbasis of Q_A

How can this be done efficiently? → **Our idea：Decoherence by measurements of Q_A**



2. Quantum Algorithm for Entanglement Asymmetry

Naively, one need to remove $\mathcal{O}(N_A^2)$ off-diagonal elements.



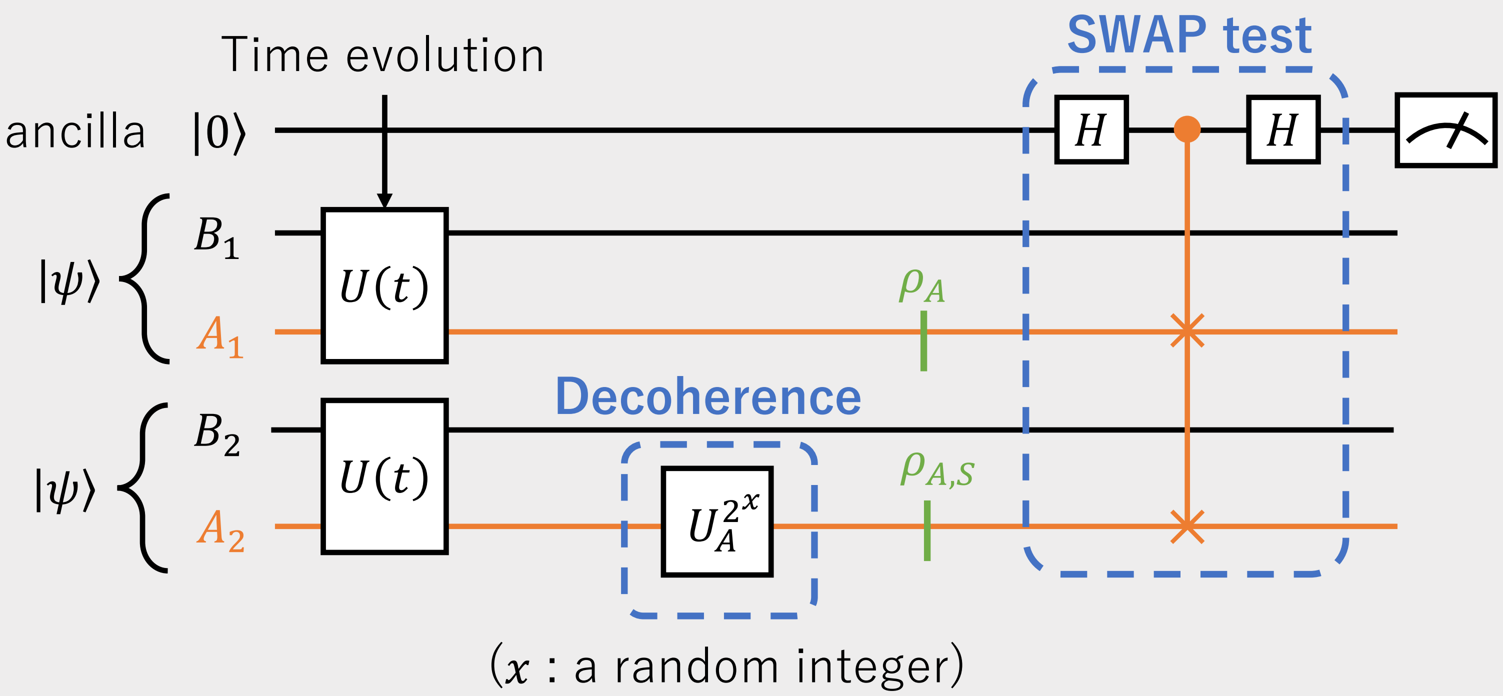
Treat this as a decoherence.

How can this be done efficiently? → **Our idea : Decoherence by measurements of Q_A**

We can drastically simplify the quantum circuit.

2. Quantum Algorithm for Entanglement Asymmetry

Overview of our quantum circuit



$\mathrm{Tr}[\rho_A^2]$ or $\mathrm{Tr}[\rho_{A,S}^2]$

$= \langle Z \rangle_{\mathrm{ancilla}}$

Statistical error $\delta \sim \frac{1}{\sqrt{N_{\mathrm{shot}}}}$

does not depend on system size

Our quantum algorithm is scalable and therefore applicable to large systems, including quantum field theories!

Outline

1. Introduction
2. Quantum Algorithm for Entanglement Asymmetry
3. Quantum Mpemba Effect in Schwinger Model
4. Summary

3. Quantum Mpemba Effect in Schwinger Model

Schwinger model in the continuum

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + \mathrm{i}gA_{\mu})\psi - m\bar{\psi}e^{\mathrm{i}\theta\gamma^5}\psi$$

Temporal gauge $A_0 = 0$

Kogut-Susskind formalism

Open boundary condition

Gauss law constraint

Jordan-Wigner transformation

Schwinger model as a spin model [Masazumi Honda et al, 2022]

$$H = H_{ZZ} + H_{\pm} + H_Z$$

$$H_{ZZ} \sim (\mathrm{const}) \sum_{n} \sum_{1 \le k < \ell \le n} Z_k Z_\ell \ , \quad H_{\pm} \sim \sum_{n} (\mathrm{const})(X_n X_{n+1} + Y_n Y_{n+1}) \ , \quad H_Z = \sum_{n} (\mathrm{const})\, Z_n$$

X_n, Y_n, Z_n : Pauli matrices on the n-th site.

3. Quantum Mpemba Effect in Schwinger Model

Details of the setup:

Symmetry

$U(1)$ symmetry, $Q = \frac{1}{2}\sum_n Z_n$

Initial state

$$|\phi\rangle = e^{-\mathrm{i}\frac{\phi}{2}\sum_n Y_n Y_{n+1}} |\uparrow\uparrow\cdots\uparrow\rangle$$

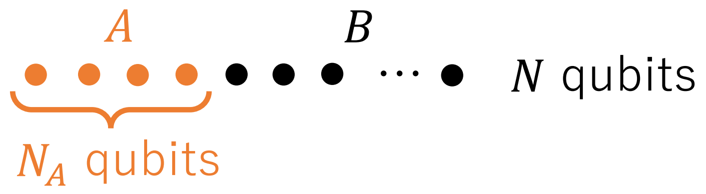
ϕ : Parameter of initial symmetry breaking.

Time evolution : Trotterization

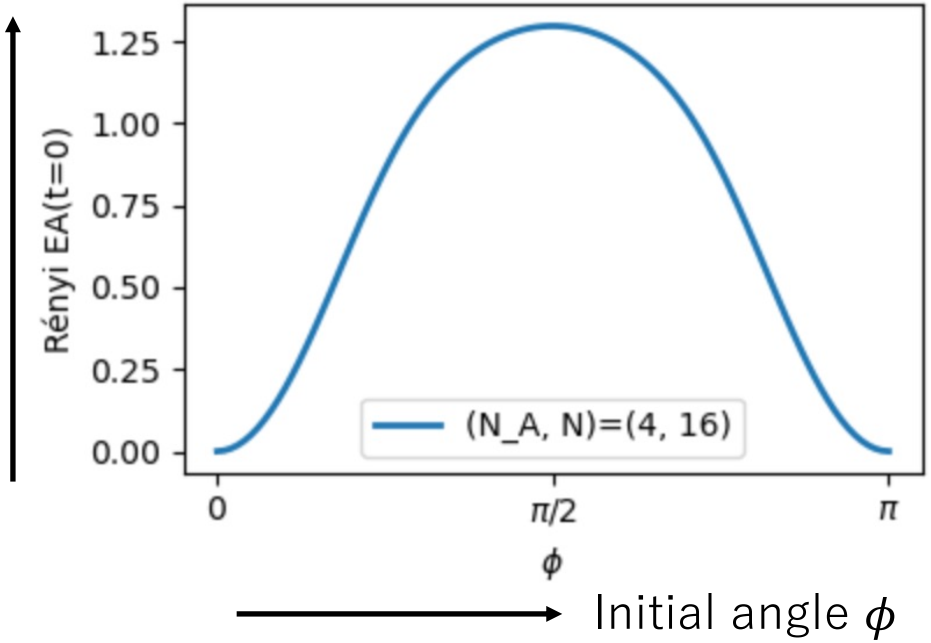
$$U(\delta t) = e^{-\mathrm{i}H_{ZZ}\delta t} \underline{e^{-\mathrm{i}H_{\pm,\mathrm{even}}\delta t} e^{-\mathrm{i}H_{\pm,\mathrm{odd}}\delta t}} e^{-\mathrm{i}H_Z \delta t}$$

$H_\pm =$ (even sites) + (odd sites)

$U(1)$ symmetric decomposition

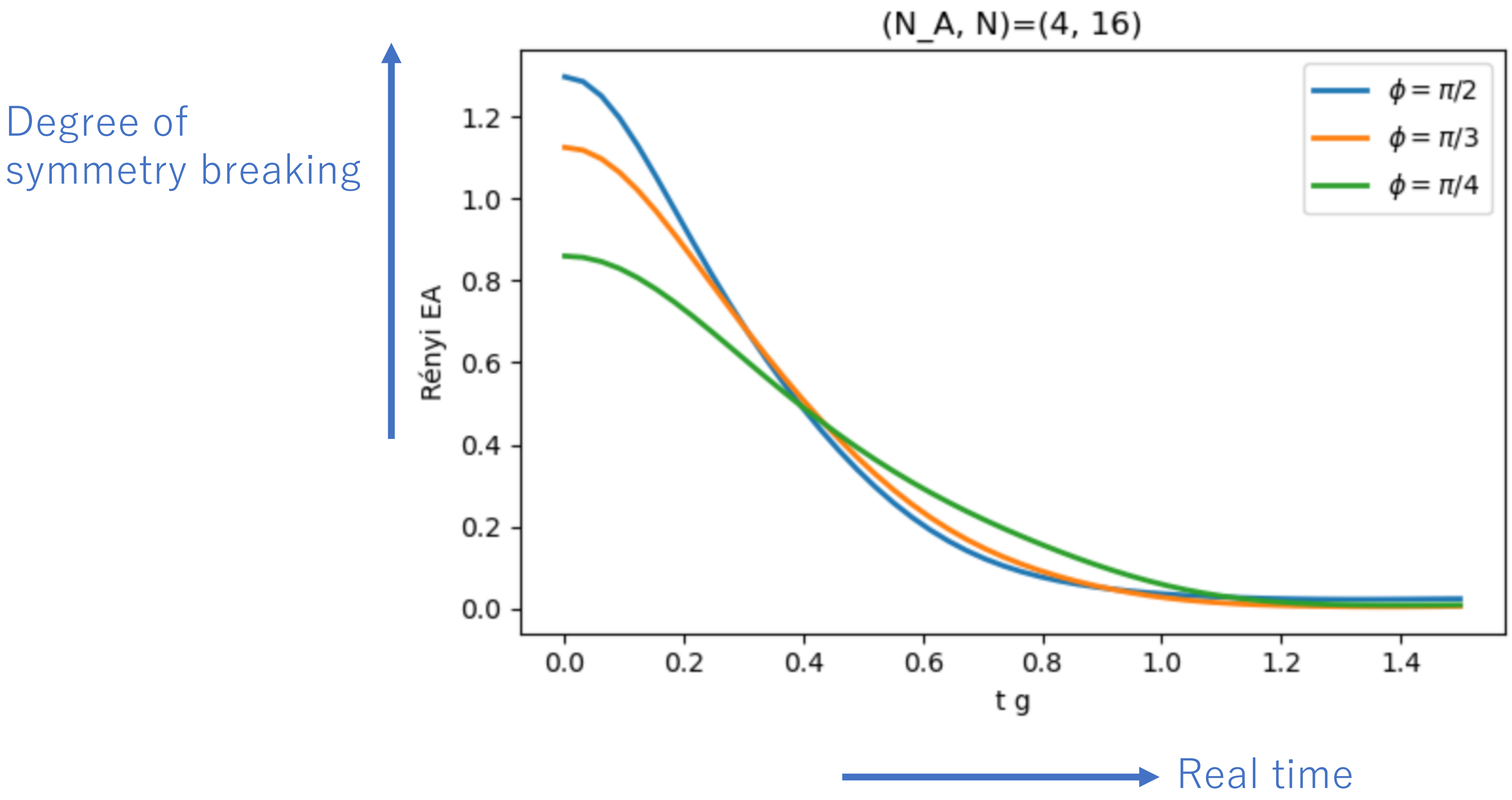


Degree of symmetry breaking at $t = 0$



3. Quantum Mpemba Effect in Schwinger Model

We performed quantum simulations using Qulacs, a Python library for classical simulation.



parameters

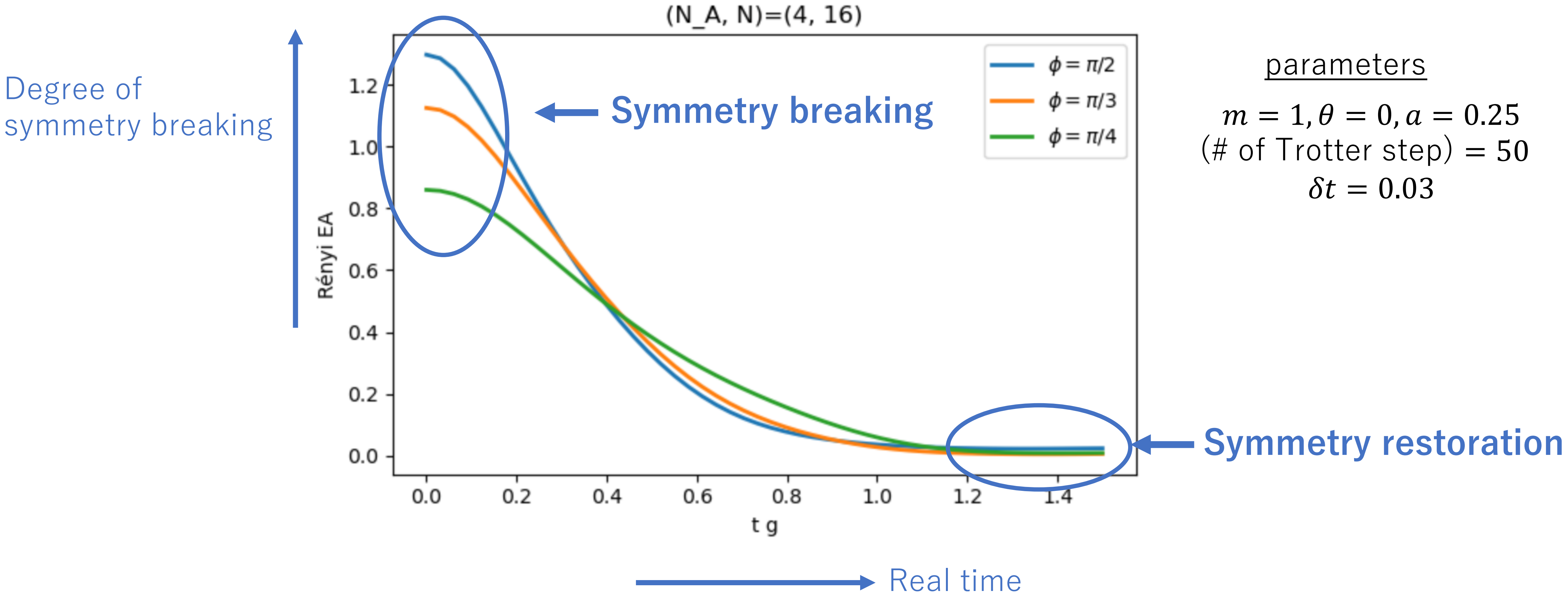
$m = 1, \theta = 0, a = 0.25$

$(\# \text{ of Trotter step}) = 50$

$\delta t = 0.03$

3. Quantum Mpemba Effect in Schwinger Model

We performed quantum simulations using Qulacs, a Python library for classical simulation.



parameters

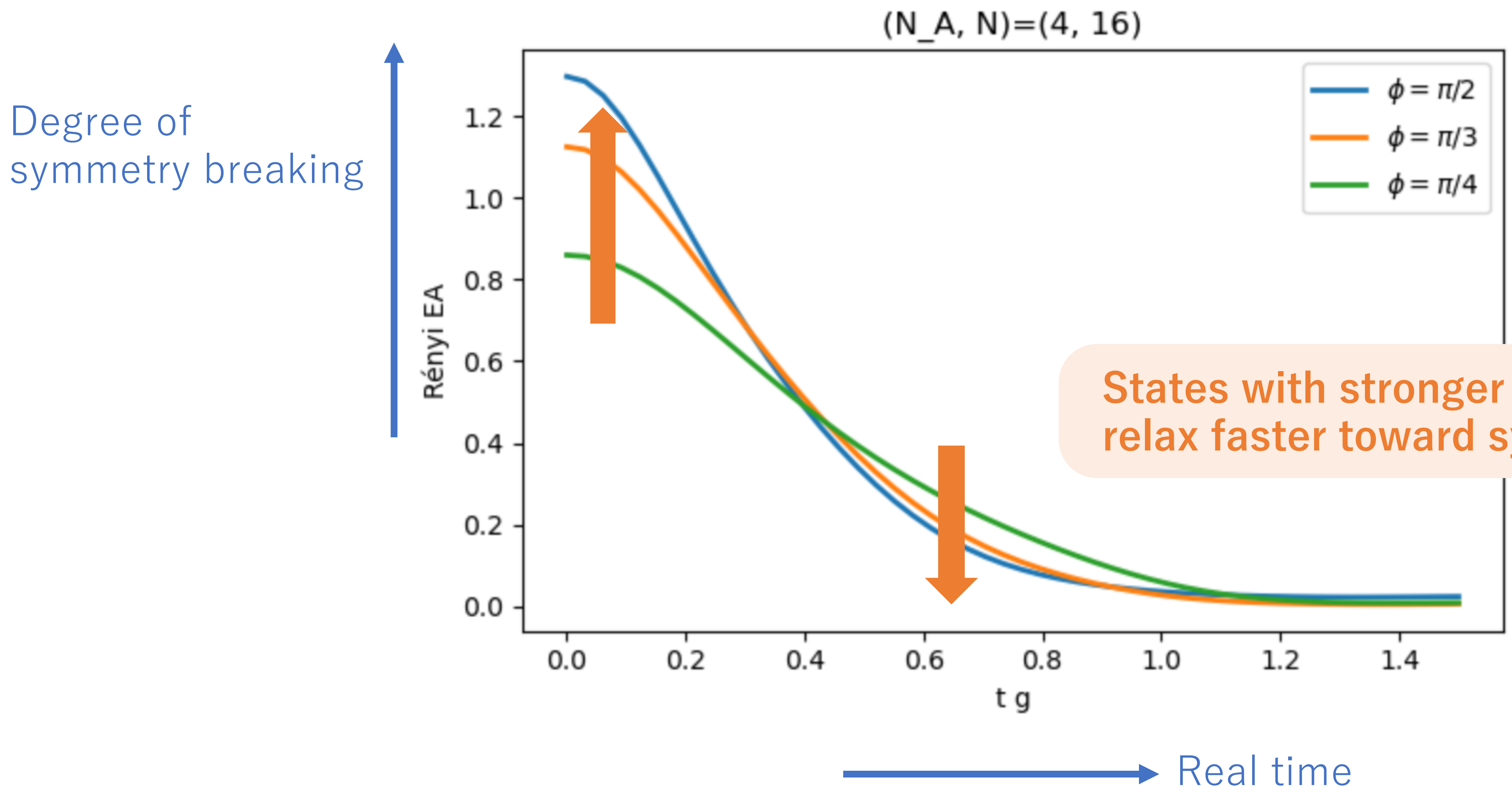
$m = 1, \theta = 0, a = 0.25$

(# of Trotter step) $= 50$

$\delta t = 0.03$

3. Quantum Mpemba Effect in Schwinger Model

We performed quantum simulations using Qulacs, a Python library for classical simulation.



<u>parameters</u>

$m = 1, \theta = 0, a = 0.25$
(# of Trotter step) $= 50$
$\delta t = 0.03$

States with stronger symmetry breaking relax faster toward symmetry restoration.

※We are currently estimating the errors.

Using our quantum algorithm, we demonstrate the quantum Mpemba effect in the Schwinger model.

Outline

1. Introduction
2. Quantum Algorithm for Entanglement Asymmetry
3. Quantum Mpemba Effect in Schwinger Model
4. Summary

1. Summary

Summary

- The quantum Mpemba effect is an anomalous out-of-equilibrium phenomenon that has been attracting attention.
- However, analyzing this phenomenon remains challenging, both analytically and numerically.
- In this work, **we propose an efficient quantum algorithm to compute the entanglement asymmetry**.
- Using this algorithm, we demonstrate the quantum Mpembe effect in the Schwinger model.

Future direction

- Error estimation for Trotterization (ongoing)
- Resource estimation (ongoing)
- Continuum limit (ongoing)
- **Applicable to other models : our algorithm works for general quantum state .**

Appendix

Appendix

Schwinger model as a spin model

$$H = H_{ZZ} + H_{\pm} + H_Z$$

$$H_{ZZ} = \frac{J}{2}\sum_{n=2}^{N-1}\sum_{1\le k<\ell\le n} Z_k Z_\ell \ , \qquad H_{\pm} = \frac{1}{2}\sum_{n=1}^{N-1}\left(w - (-1)^n \frac{m}{2}\sin\theta\right)(X_n X_{n+1} + Y_n Y_{n+1})$$

$$H_Z = \frac{m\cos\theta}{2}\sum_{n=1}^{N}(-1)^n Z_n - \frac{J}{2}\sum_{n=1}^{N-1}(n \bmod 2)\sum_{\ell}^{n} Z_\ell \ , \quad J = \frac{g^2 a}{2}, w = \frac{1}{2a}, a : \text{lattice spacing}$$