Nonstabilizerness Mpemba Effects

Harunobu Fujimura

2026/6/17 12:10-13:00

Based on arXiv:2605.04155

Zhenyu Xiao, Hao-Kai Zhang, Shuo Liu

1. Introduction

Today's topic

Physics ⇄ **Quantum Computing**

Science — Engineering

Although these fields are often viewed as distinct disciplines, they are deeply connected both **practically** and **conceptually.**

1. Introduction

Today's topic

conceptual

Physics ⇄ **Quantum Computing**

Science | Engineering

In this talk, I will discuss how a concept originating from physics can provide new insights into quantum computing.

1. Introduction

Today's topic

conceptual

Mpemba effect
Anomalous relaxation in nonequilibrium physics

Nonstabilizerness
A resource for quantum computation

Main question:

Can a phenomenon discovered in physics help quantum computing?

The answer is **yes** (This work).

1. Introduction

Today's topic

conceptual

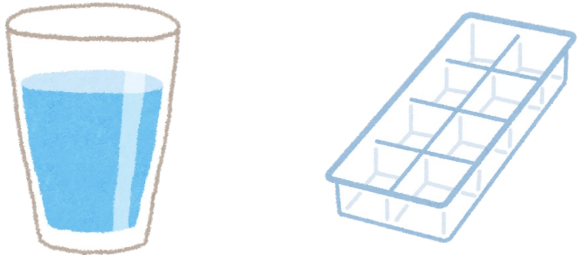
Mpemba effect
Anomalous relaxation in nonequilibrium physics

Nonstabilizerness
A resource for quantum computation

What is the Mpemba effect?

1. Introduction

Consider the following question: How can we make ice faster?



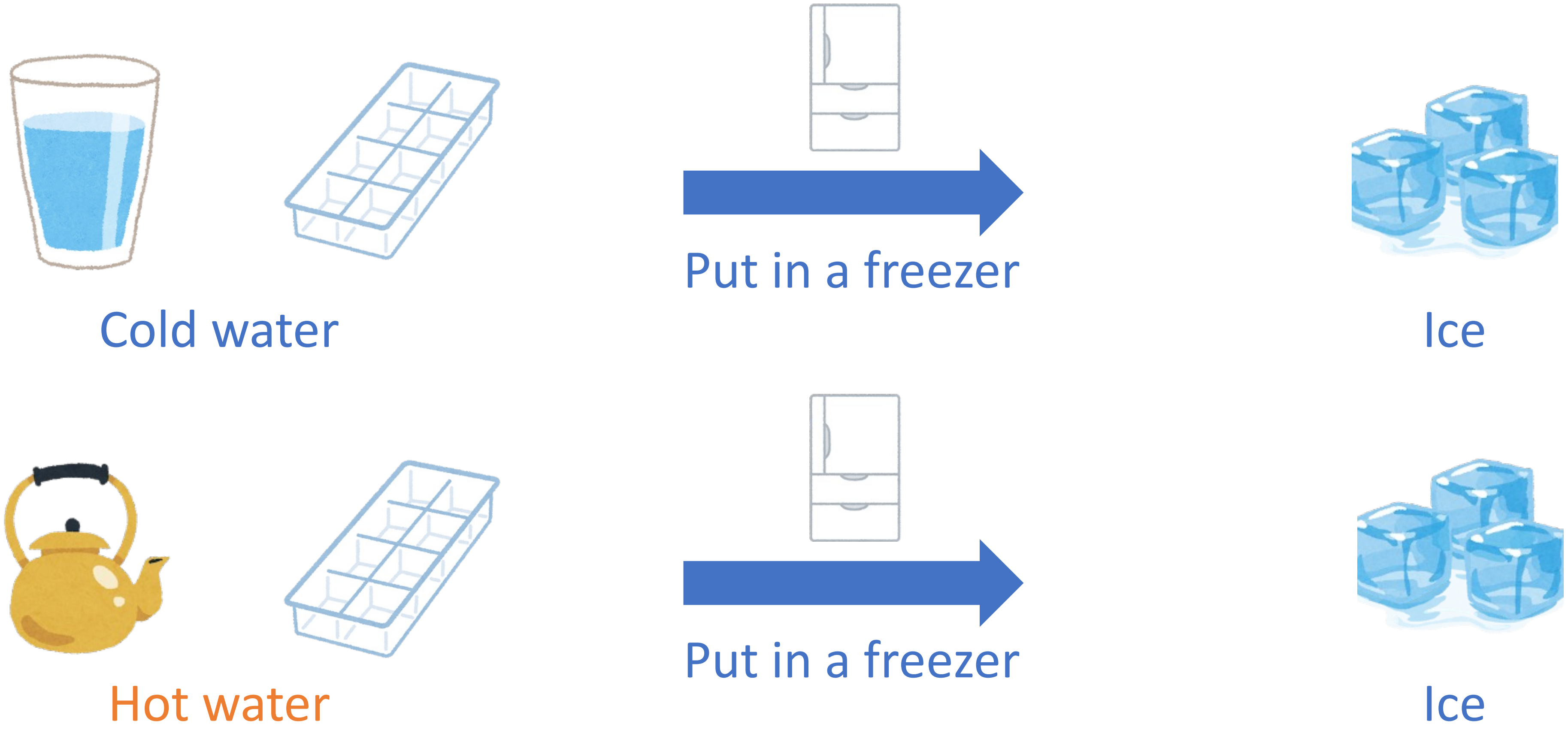
Cold water

Put in a freezer

Ice

1. Introduction

Consider the following question: How can we make ice faster?

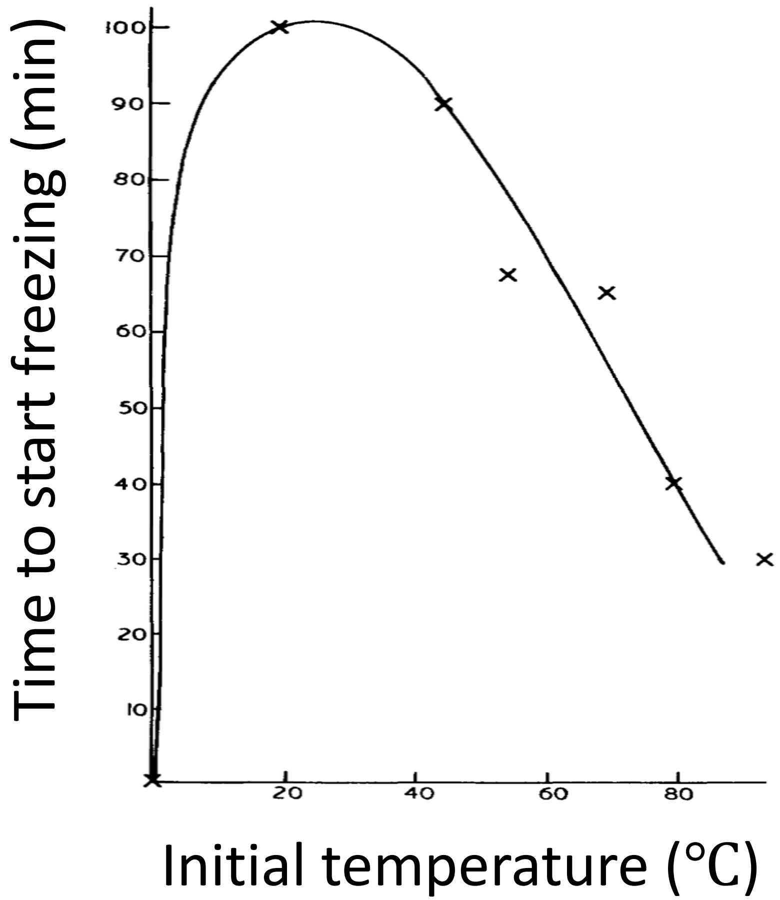


Intuitively, one expect cold water to freeze faster than hot water.

1. Introduction

Mpemba and Osborne conduct an experiment and a surprising phenomenon was observed.

[E B Mpemba and D G Osborn, 1969]



1. Introduction

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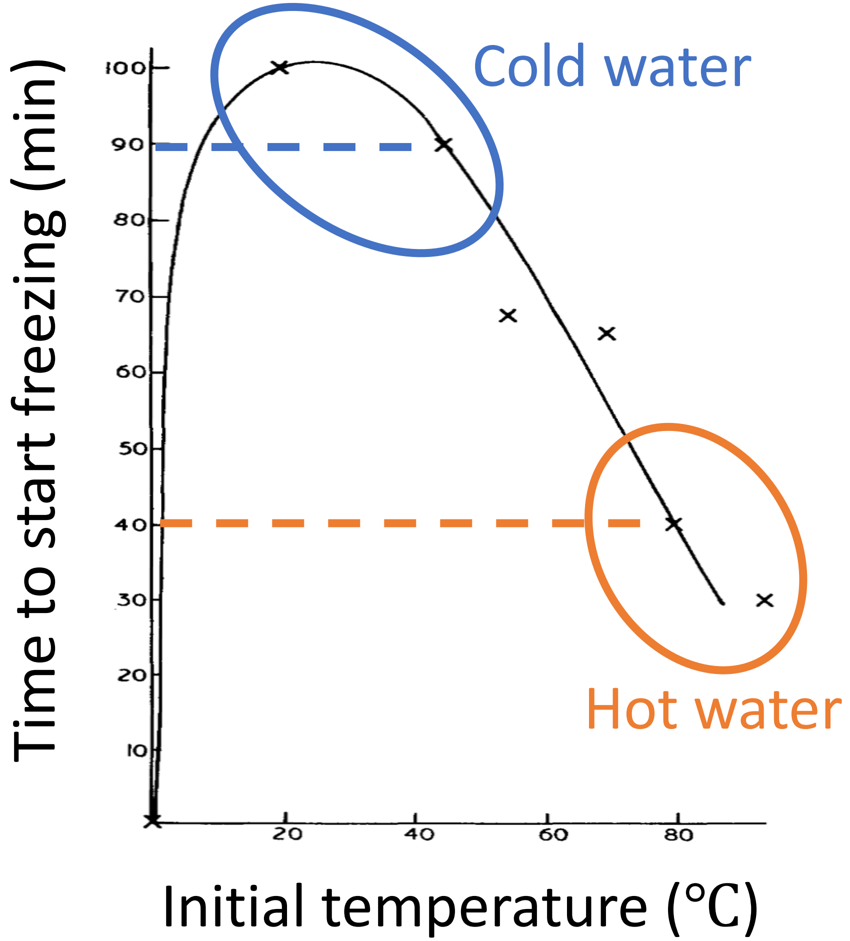


Hot water can freeze faster than cold water!

1. Introduction

Question:
How can we make ice faster?

Answer:
Use hot water instead of cold water.

Source: Science-tech/2023-03-25

This is the Mpemba effect:
an anomalous relaxation in nonequilibrium physics

1. Introduction

The Mpemba effect was first observed in classical systems, but a quantum counterpart has been found.

[Ares-Murciano-Calabrese, 2022]

Symmetry-broken state

Quench with a symmetric Hamiltonian

Symmetry restoration

Initial state 1

(Weakly symmetry-broken)

1. Introduction

The Mpemba effect was first observed in classical systems, but a quantum counterpart has been found.

[Ares-Murciano-Calabrese, 2022]

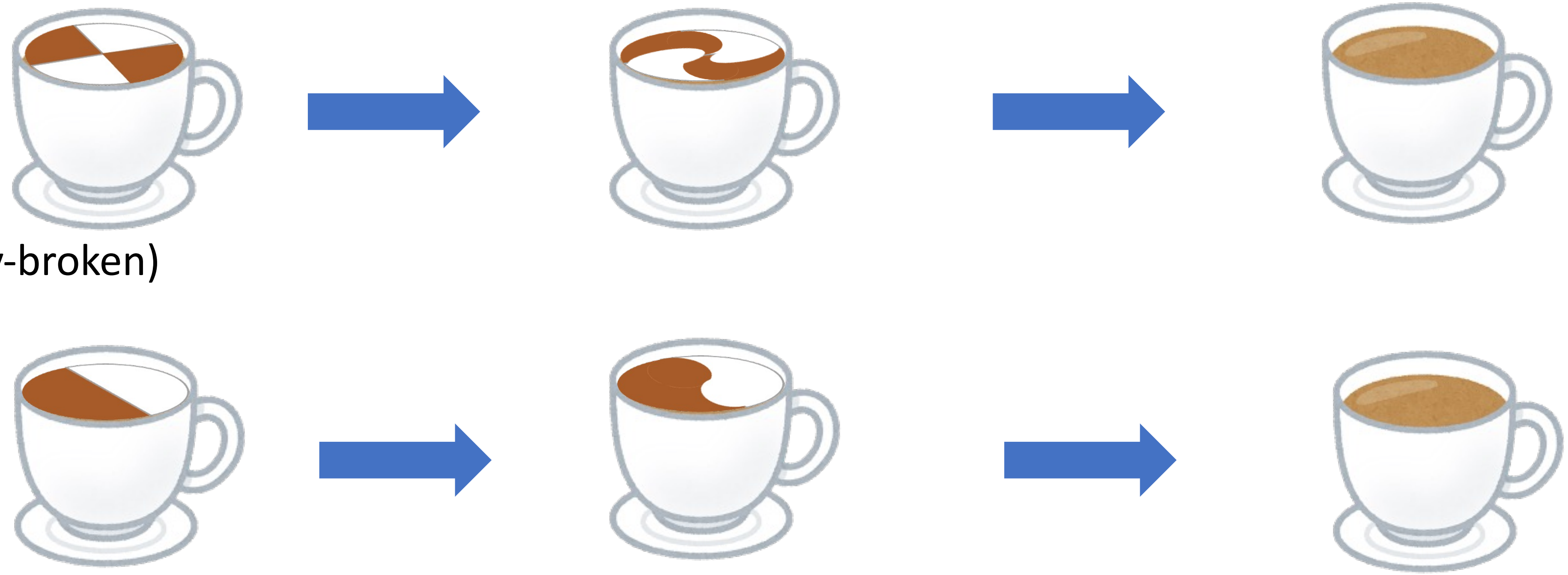
Symmetry-broken state | **Quench with a symmetric Hamiltonian** | **Symmetry restoration**

Initial state 1

(Weakly symmetry-broken)

Initial state 2

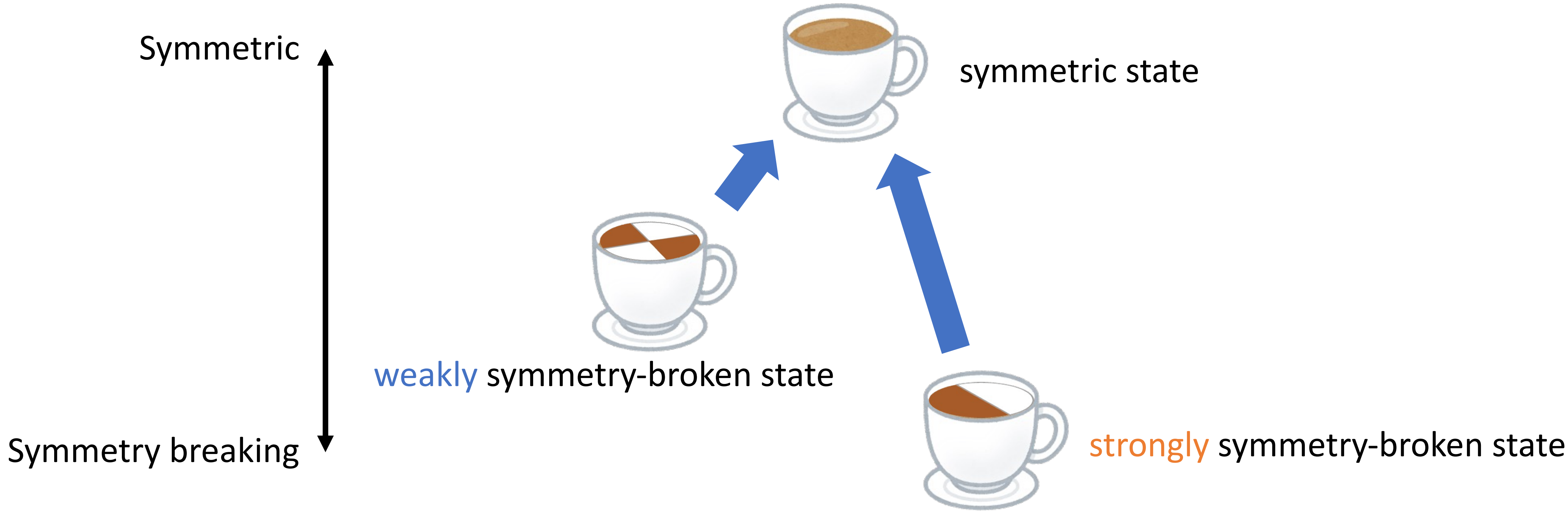
(Strongly symmetry-broken)



Question: Which state relaxes faster to the symmetric state?

1. Introduction

Intuitively, one expects the weakly symmetry-broken state to relax faster, since it is initially closer to the symmetric state.



Surprisingly, the opposite can happen.

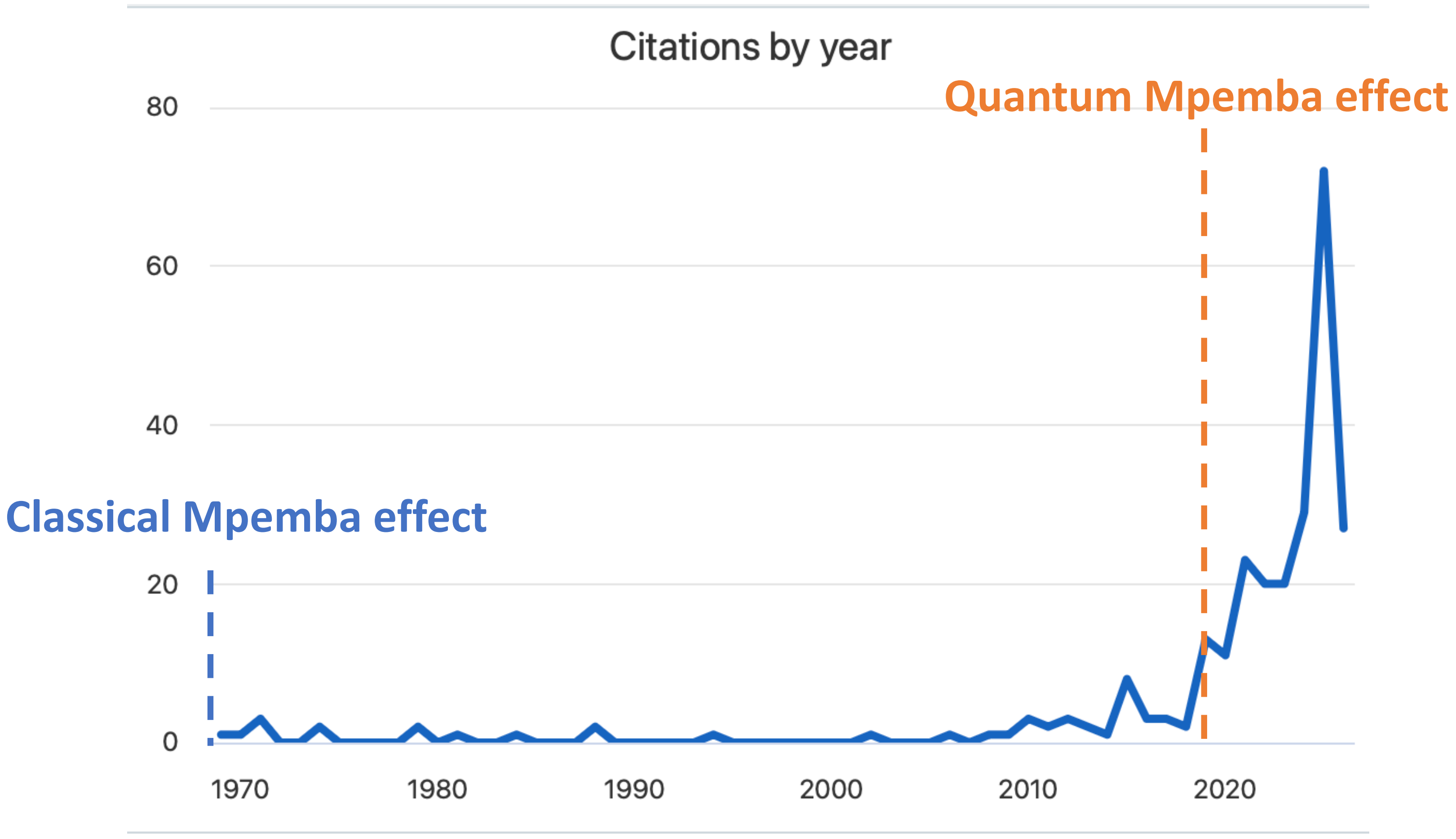
Counterintuitive!

Quantum Mpemba effect

The more symmetry is initially broken, the faster it is restored.

1. Introduction

The Mpemba effect has attracted considerable attention.



Research on the Mpemba effect has grown rapidly!

1. Introduction

Today's topic

Mpemba effect

Anomalous relaxation in nonequilibrium physics

conceptual

Nonstabilizerness

A resource for quantum computation

Next, I will briefly explain this.

1. Introduction

What is the quantum computing?

Classical computing

Unit of information: bit $\boxed{0}$ or $\boxed{1}$

Data: N-bit string

$$\boxed{0}\ \boxed{1}\ \boxed{0}\ \cdots \quad = |010\cdots\rangle$$

Manipulates all 2^N basis states individually.

Quantum computing

Unit of information: qubit  $= \alpha|0\rangle + \beta|1\rangle$

Data: N-qubit state

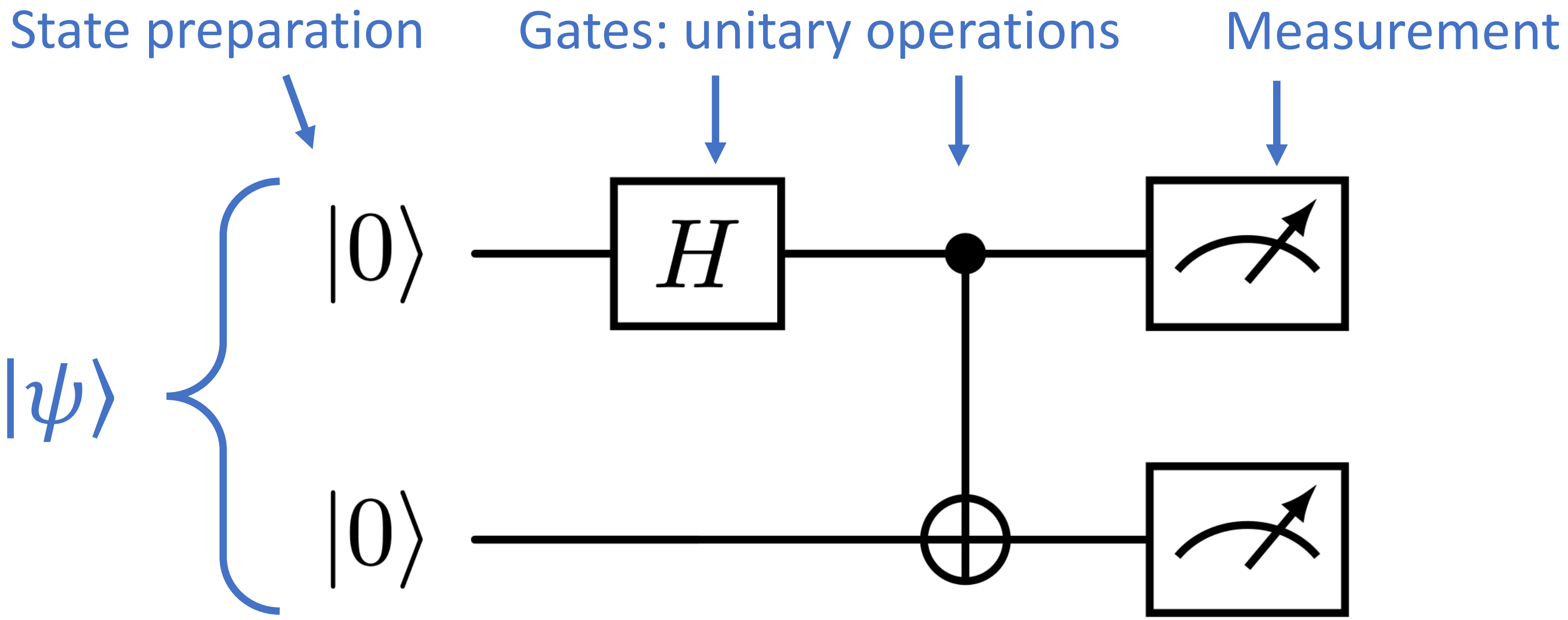
 $\cdots \ = \sum_{i_1,\cdots i_N=0}^{1} c_{i_1,\cdots i_N} |i_1 \cdots i_N\rangle$

Manipulates a **superposition** of exponentially many basis states.

Quantum computing harnesses quantum mechanics for computation.

1. Introduction

How do we perform a quantum computation?

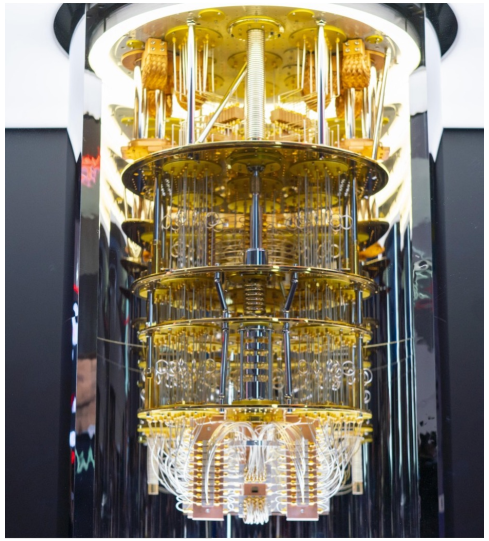


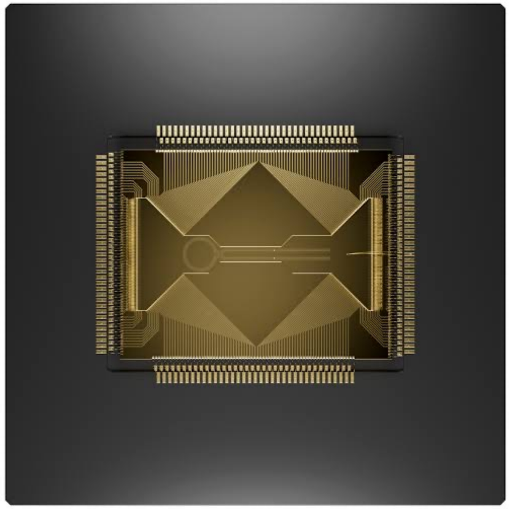
"Quantum Circuit"

Quantum states are manipulated by a sequence of quantum gates.
Information is extracted through measurements.

1. Introduction

Many research institute and industries are developing quantum computers.


IBM


Quantinuum

Riken, Fujitsu

and so on…

Quantum computers are becoming a real-world technology.

1. Introduction

Central question:

What makes quantum computers powerful?

1. Introduction

Central question:
What makes quantum computers powerful?

A large number of qubits is not enough.

A simple example

Consider a quantum computer with **100-qubits**

Input: $$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes 100} + |1\rangle^{\otimes 100}\right)$$

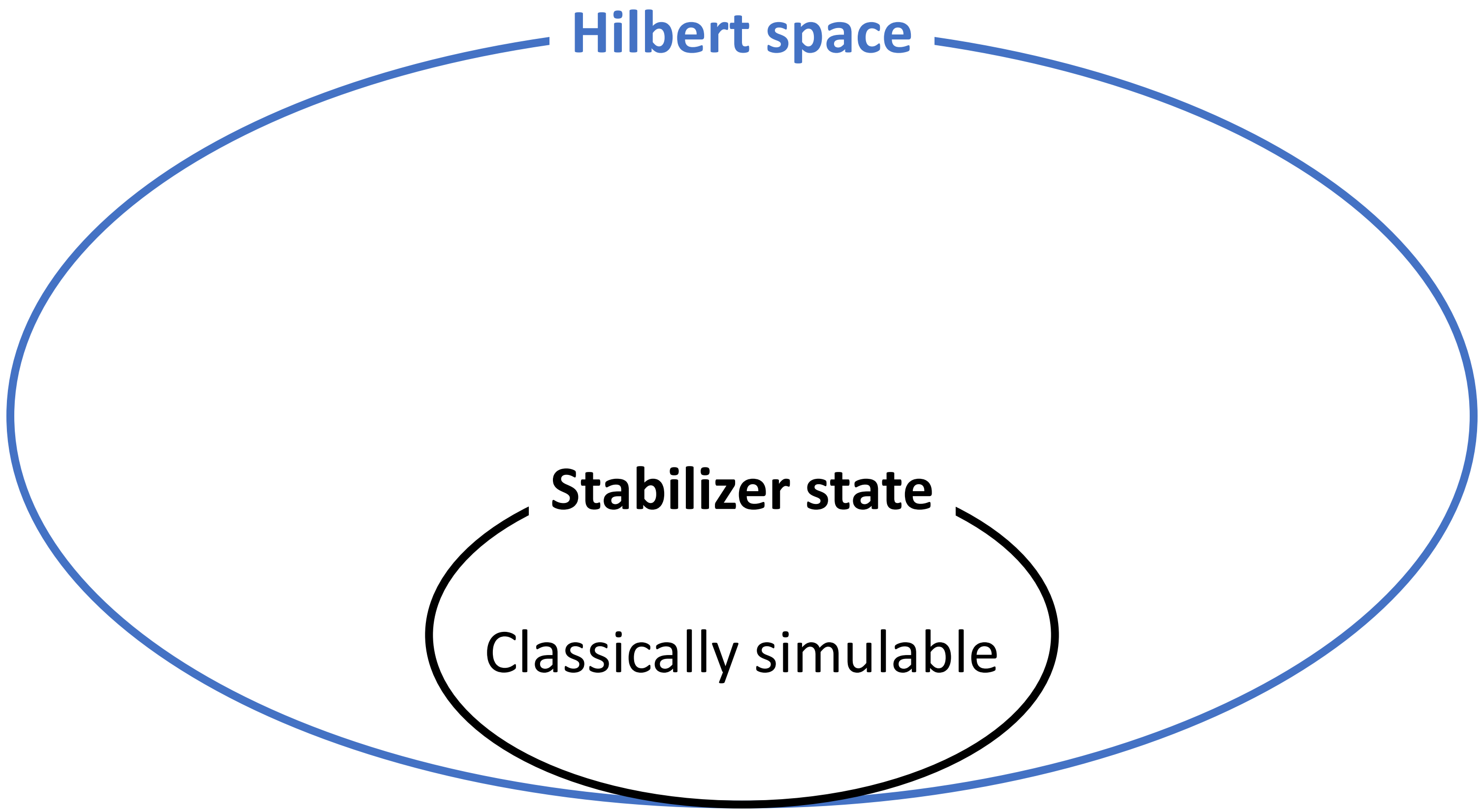
Output: $\langle\psi|H|\psi\rangle$, where H is spanned by $\{|0\rangle^{\otimes 100}, |1\rangle^{\otimes 100}\}$

This is effectively just a 2×2 matrix computation!
(Classical computer can simulate this easily!)

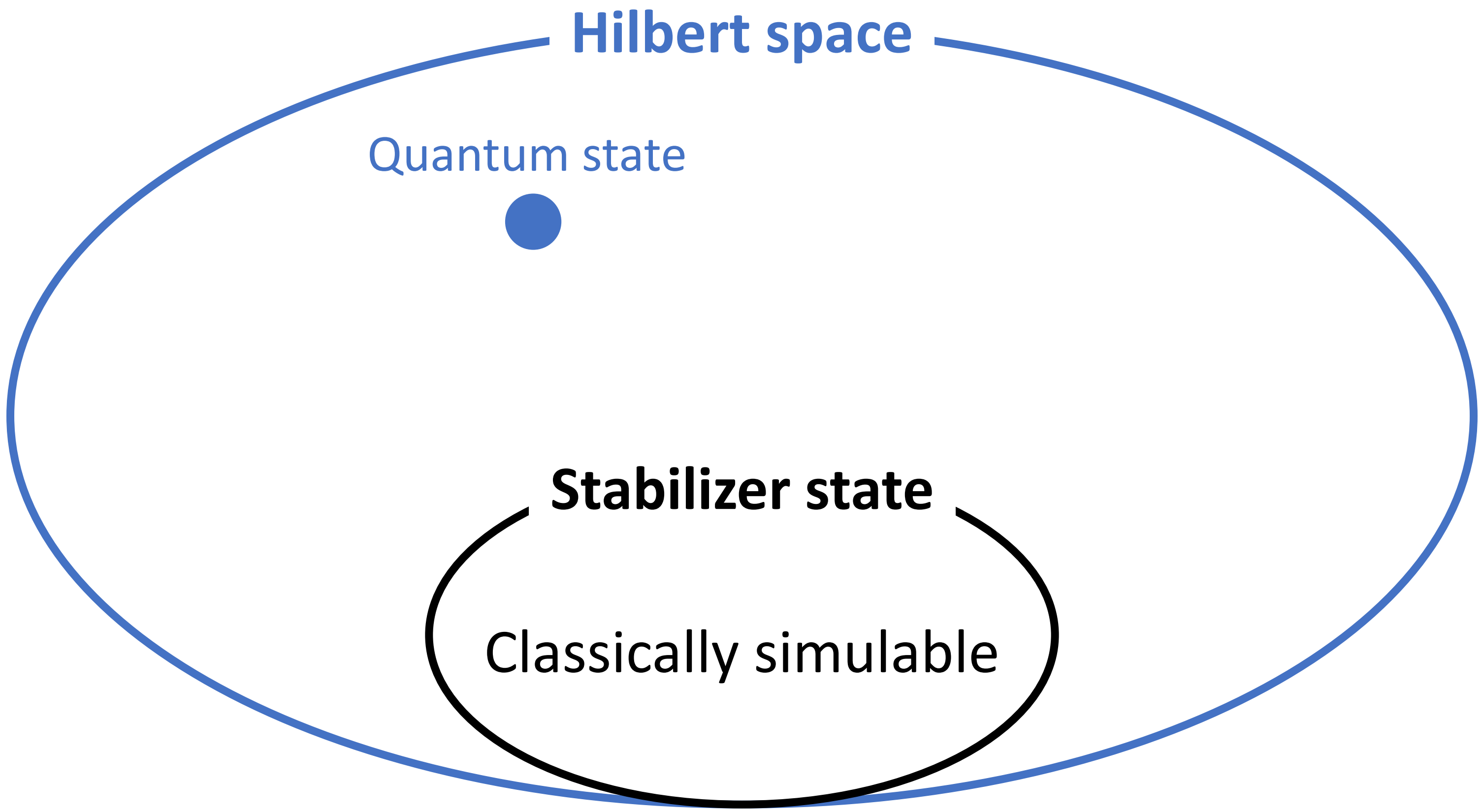
✗ **"How many qubits?"**

○ **"What kind of quantum state?"**

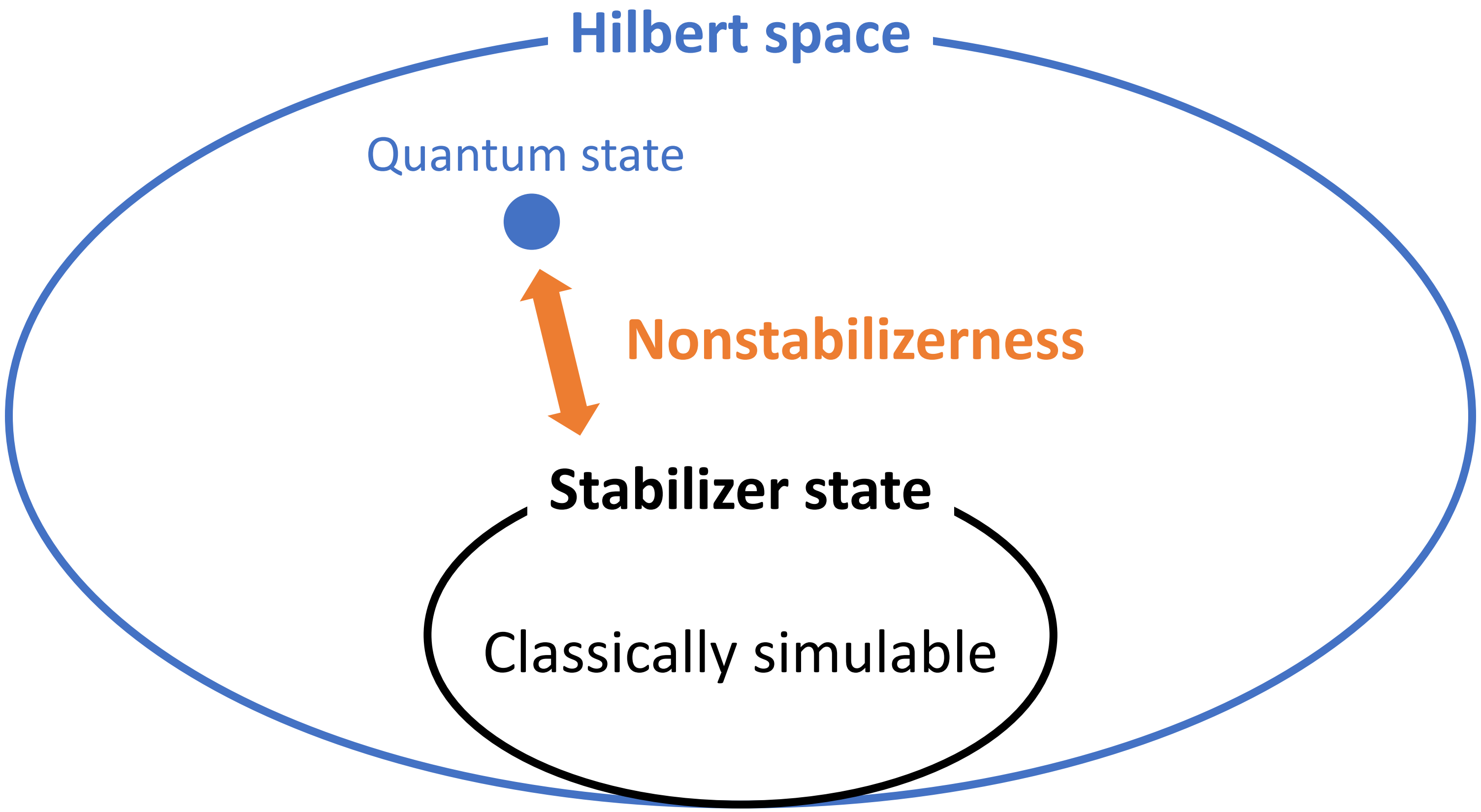
1. Introduction



1. Introduction



1. Introduction



Nonstabilizerness is a resource for quantum advantage.

1. Introduction

We need quantum states with large nonstabilizerness to achieve quantum advantage.
How can we prepare such states efficiently?

Classical Mpemba Effect
Hot water can freeze faster than cold water.

Quantum Mpemba Effect
A more strongly symmetry-broken state relaxes faster toward the symmetric state.

1. Introduction

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Quantum Mpemba Effect
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↓

Nonstabilizerness Mpemba Effect (This work)
A state with less initial nonstabilizerness can generate nonstabilizerness faster.

Outline

1. Introduction

2. Quantum Computing and Nonstabilizerness

3. Nonstabilizerness Mpemba Effects

4. Summary

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4. Summary

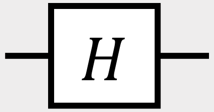
2. Quantum Computing and Nonstabilizerness

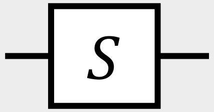
One of the most widely used universal gate set is "**Clifford + T**"

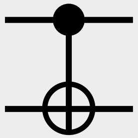
Let $\mathcal{P}_N$ be N-qubit Pauli group: $\mathcal{P}_N = \{\pm 1, \pm i\} \times \{I, X, Y, Z\}^{\otimes N}$, where X, Y, Z are Pauli matrices.

Clifford gates

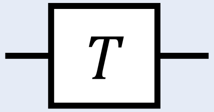
$$U \text{ is a Clifford gate} \overset{\text{def}}{\Leftrightarrow} \forall P \in \mathcal{P}_N, UPU^\dagger \in \mathcal{P}_N$$

Hadamard gate  $= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

S gate  $= \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

CNOT gate  $= |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes I$

T gate

 $= \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

T gate is no-Clifford gate:

e.g.

$$TXT^\dagger = \frac{X+Y}{\sqrt{2}} \notin \mathcal{P}_N$$

Easy to implement fault-tolerantly

Expensive to implement

2. Quantum Computing and Nonstabilizerness

What is the stabilizer state?

Stabilizer group

$$S \subset \mathcal{P}_N,\ \forall S_i, S_j \in S, \left[S_i, S_j\right] = 0$$

e.g. $S = \{I, Z_1, Z_2, Z_1 Z_2\}$

Commuting subgroup of the Pauli group

Stabilizer state

For any $S_i \in S$: Stabilizer group, s.t. $|S| = 2^N$

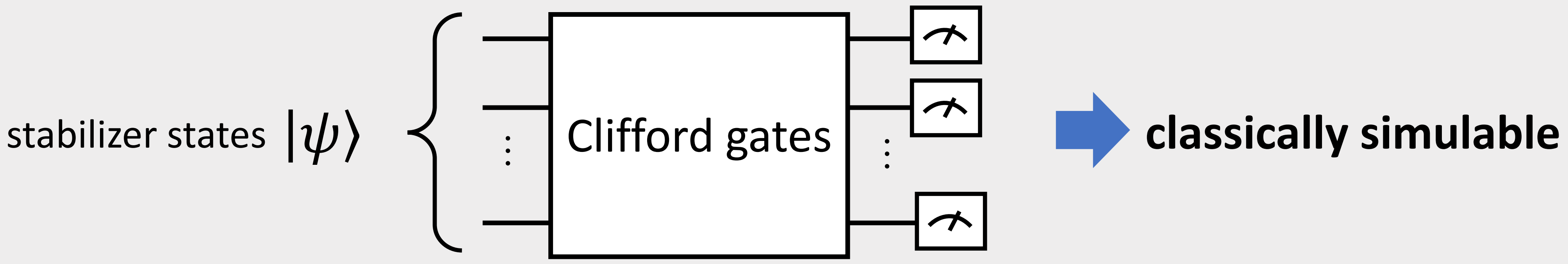
$$S_i|\psi\rangle = |\psi\rangle$$

e.g. $|00\rangle \ \rightarrow Z_1|00\rangle = Z_2|00\rangle = Z_1 Z_2|00\rangle = |00\rangle$

Common eigenstates of Stabilizer operators

2. Quantum Computing and Nonstabilizerness

Gottesman-Knill theorem [Daniel Gottesman, 1998]



"Clifford circuits acting on stabilizer states can be simulated efficiently on a classical computer."

Quantum advantage requires "Nonstabilizerness"

2. Quantum Computing and Nonstabilizerness

How can we quantify nonstabilizerness?

First, consider the Pauli expansion of a pure state:

$$\rho = |\psi\rangle\langle\psi| = \frac{1}{2^N}\sum_{P\in\mathcal{P}_N} c_P P \;, c_P = \langle\psi|P|\psi\rangle \text{ :coefficient}$$

Stabilizer Rényi Entropy (SRE) [Lorenzo Leone et al 2021]

$$M_\alpha(|\psi\rangle) = \frac{1}{1-\alpha}\log_2\left[\sum_{P\in\mathcal{P}_N}\left(\frac{|c_P|^2}{2^N}\right)^\alpha\right] - N$$

α: Rényi index

The Stabilizer Rényi Entropy(SRE) is a quantitative measure of nonstabilizerness.

2. Quantum Computing and Nonstabilizerness

Examples of Stabilizer Rényi Entropy(SRE).

Example 1: Stabilizer state

$c_P = \langle\psi|P|\psi\rangle = 1$ if $P \in S$, otherwise, $c_P = 0$ ⇒ $\boxed{M_2 = 0}$

Nonstabilizerness = 0

2. Quantum Computing and Nonstabilizerness

Examples of Stabilizer Rényi Entropy(SRE).

Example 1: Stabilizer state

$c_P = \langle\psi|P|\psi\rangle = 1$ if $P \in S$, otherwise, $c_P = 0$ ➡ $\boxed{M_2 = 0}$

Nonstabilizerness = 0

Example 2: magic state (1-qubit)

$|T\rangle = T|+\rangle$: (non-Clifford state)

$\langle T|X|T\rangle = \langle T|Y|T\rangle = 1/\sqrt{2}$, $\langle T|Z|T\rangle = 0$ ➡ $\boxed{M_2 = \log_2\frac{4}{3} > 0}$

Positive nonstabilizerness

SRE quantifies the degree of nonstabilizerness.

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1. Introduction
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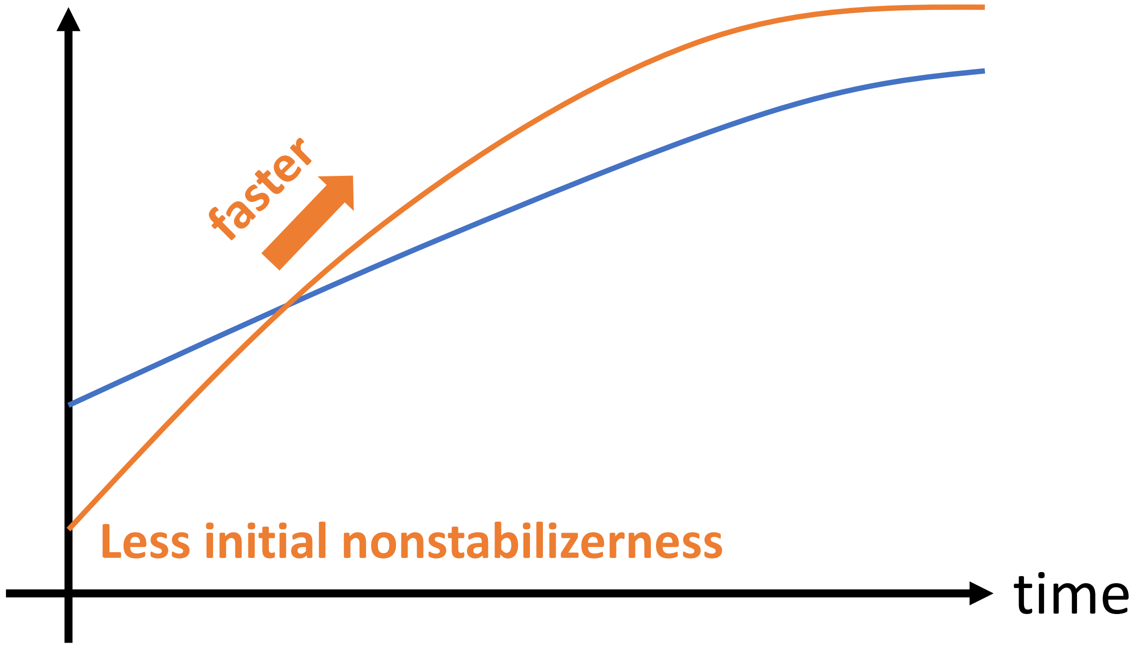
3. Nonstabilizerness Mpemba Effects

Nonstabilizerness Mpemba Effect

A state with less initial nonstabilizerness can generate nonstabilizerness faster.

What do we want to show?

SRE = Nonstabilizerness = Resource of quantum advantage

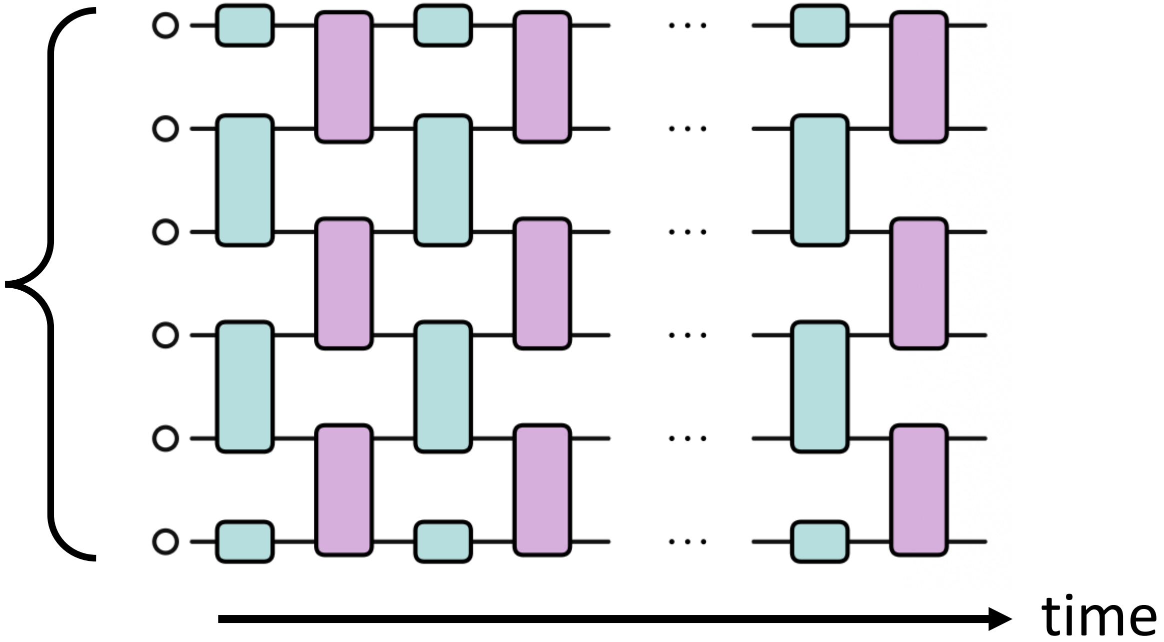


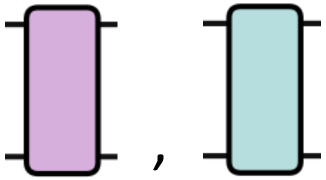
3. Nonstabilizerness Mpemba Effects

In this work, the authors consider the following quantum circuit.

Brickwork random circuit

Initial state $|\psi(0)\rangle$





= Random nearest-neighbor two-qubit unitaries, which conserves the $U(1)$ charge $Q = \sum_i n_i = \sum_i \frac{1-Z_i}{2}$

3. Nonstabilizerness Mpemba Effects

Initial states:

Tilted Ferromagnetic States (TFS): $|\psi(\theta)\rangle_{\mathrm{TFS}} = e^{-i\frac{\theta}{2}\sum_{j=1}^{N} Y_j} \underbrace{|000\cdots 0\rangle}_{Q=0}$, θ: parameter

Tilted Néel States (TNS): $|\psi(\theta)\rangle_{\mathrm{TNS}} = e^{-i\frac{\theta}{2}\sum_{j=1}^{N} Y_j} \underbrace{|0101\cdots 01\rangle}_{Q=N/2}$

Tilted Domain-Wall States (TDWS): $|\psi(\theta)\rangle_{\mathrm{TDWS}} = e^{-i\frac{\theta}{2}\sum_{j=1}^{N} Y_j} \underbrace{|0\cdots 01\cdots 1\rangle}_{Q=N/2}$

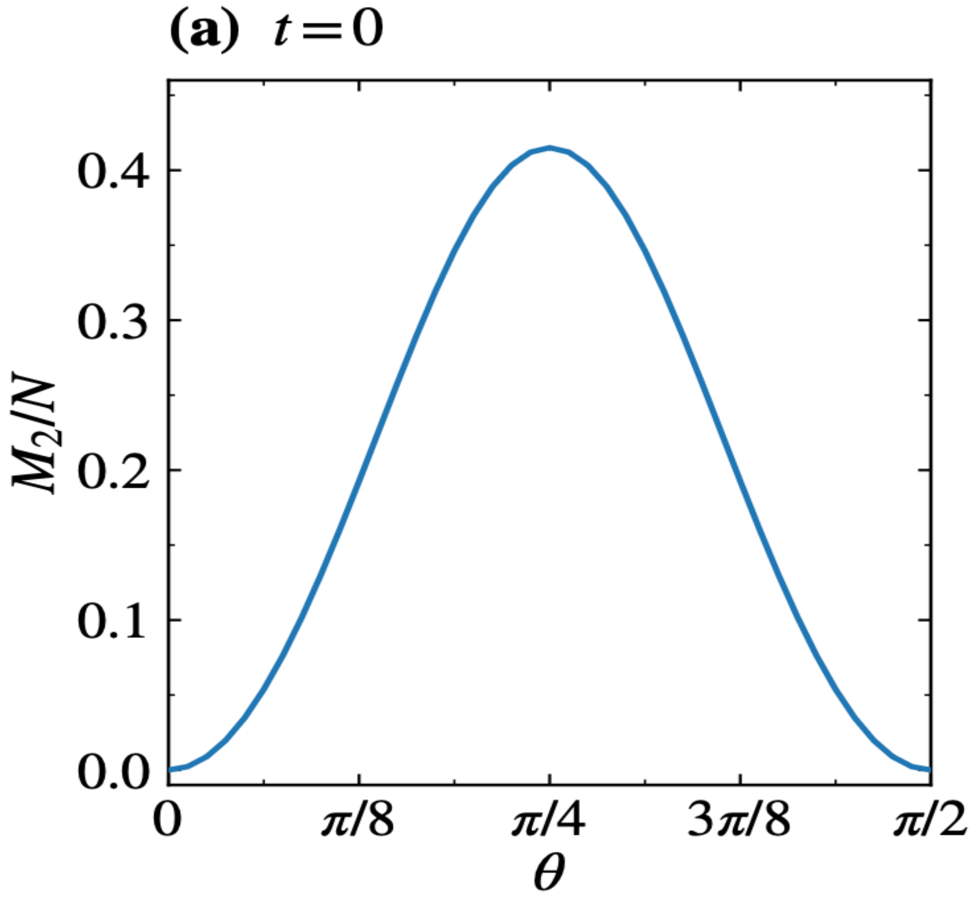
All three families are constructed from different reference states:

$|000\cdots 0\rangle,\ |0101\cdots 01\rangle,\ |0\cdots 01\cdots 1\rangle$

3. Nonstabilizerness Mpemba Effects

Initial nonstabilizeress of TFS

Stabilizer Rényi Entropy (SRE) at initial time

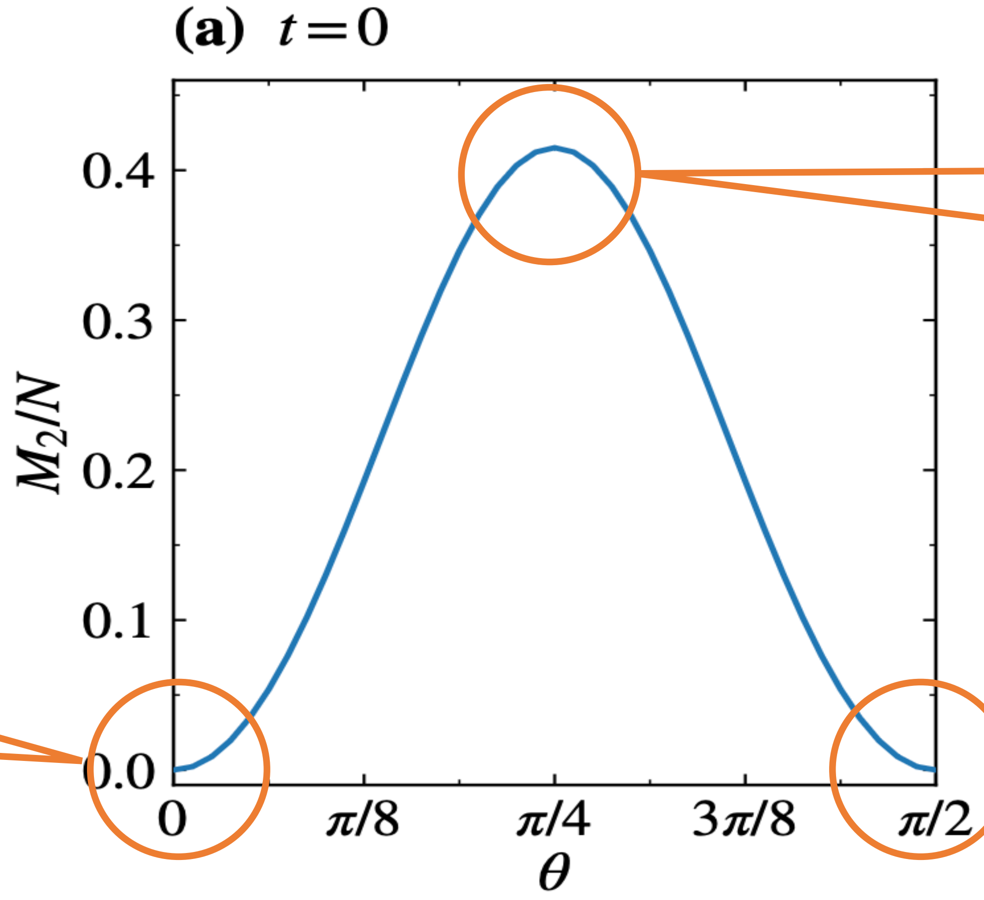


3. Nonstabilizerness Mpemba Effects

Initial nonstabilizeress of TFS

※The same behavior is observed for TNS and TDWS.

Stabilizer Rényi Entropy (SRE) at initial time



$|00\cdots0\rangle$

$\otimes_{j=1}^{N}\left(\cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle\right)$

Nonstabilizer state

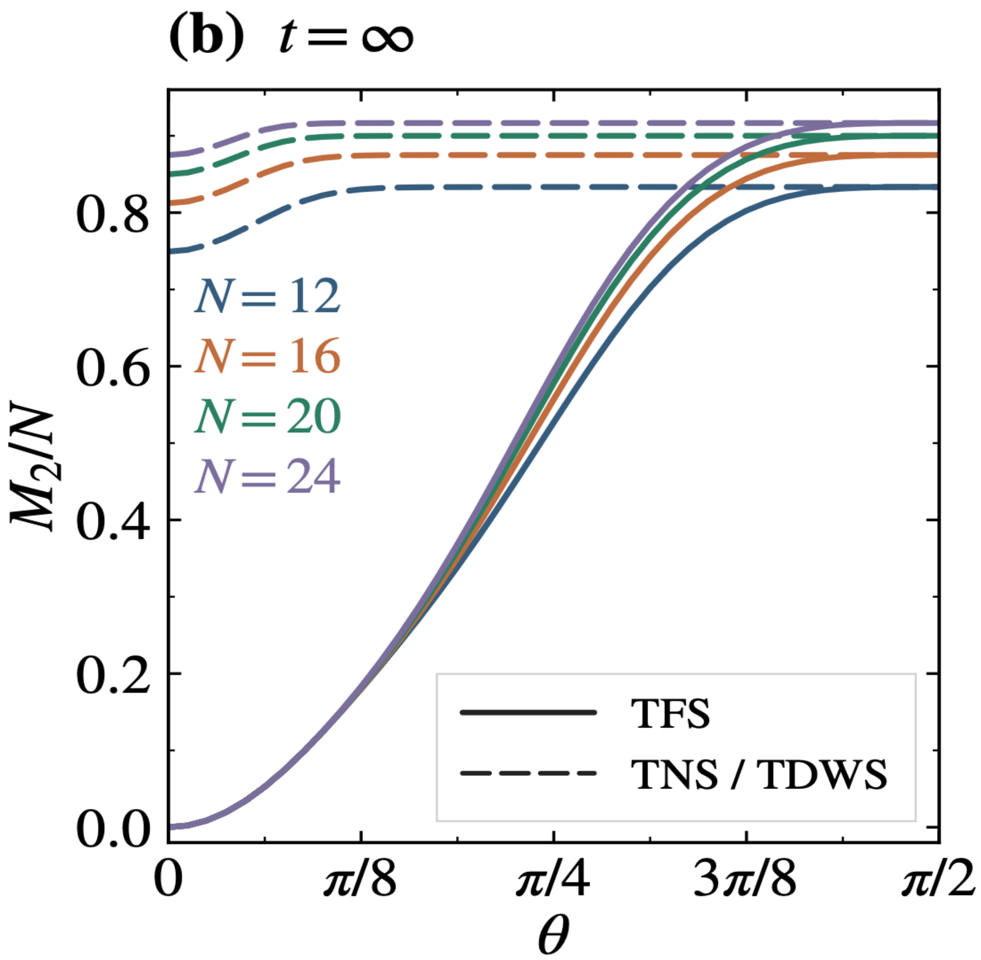
$|++\cdots+\rangle$

Initial nonstabilizerness depends strongly on the angle θ.

3. Nonstabilizerness Mpemba Effects

Late time nonstabilizerness

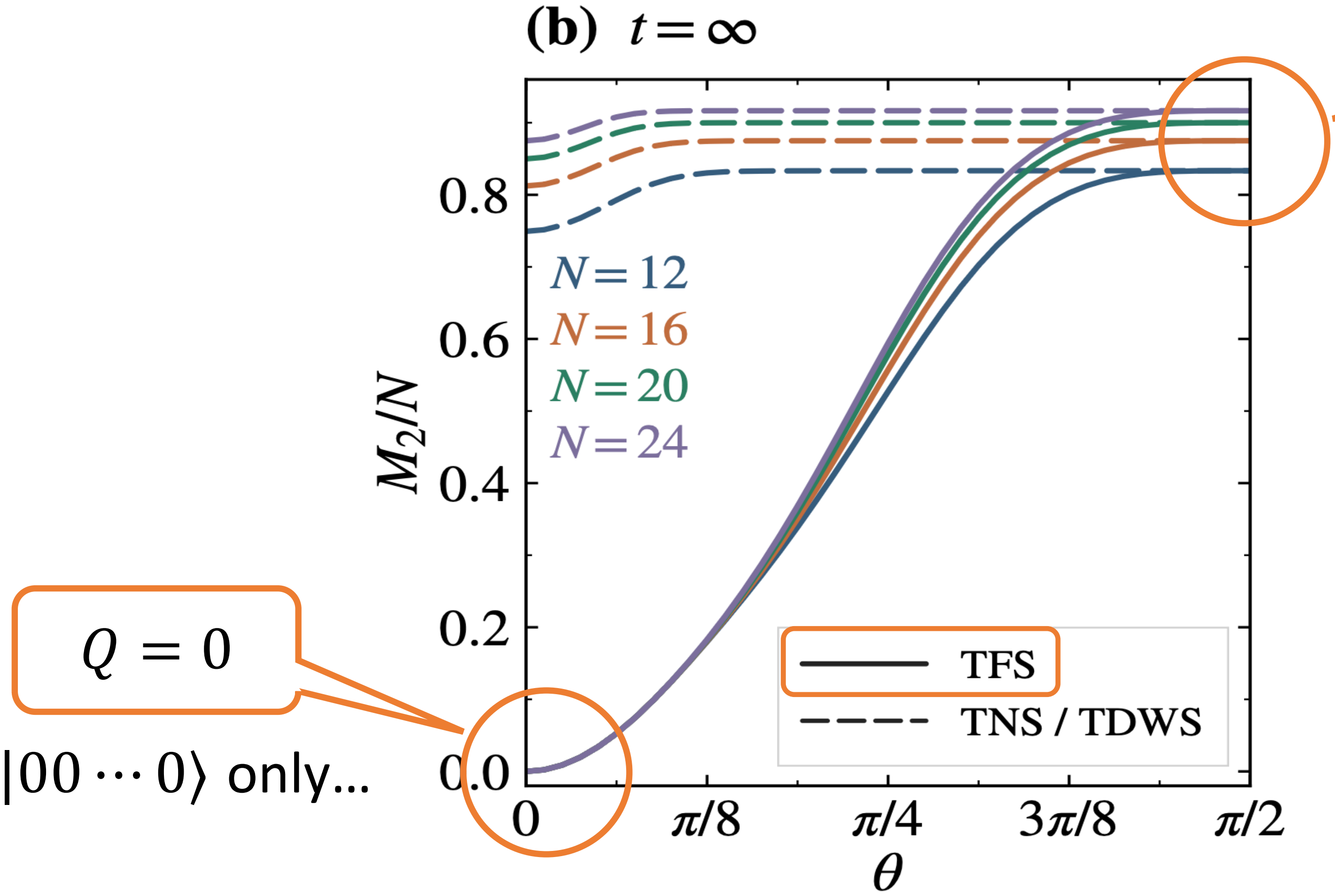
Late-time behavior of SRE:



3. Nonstabilizerness Mpemba Effects

Late time nonstabilizerness

Late-time behavior of SRE:



$|00\cdots0\rangle$ only...

$|0011\cdots\rangle, |0101\cdots\rangle,$
$|0110\cdots\rangle, |1100\cdots\rangle$

Many states

Charge-conserving random circuit ➡ **The initial charge distribution significantly affects the late-time SRE.**

3. Nonstabilizerness Mpemba Effects

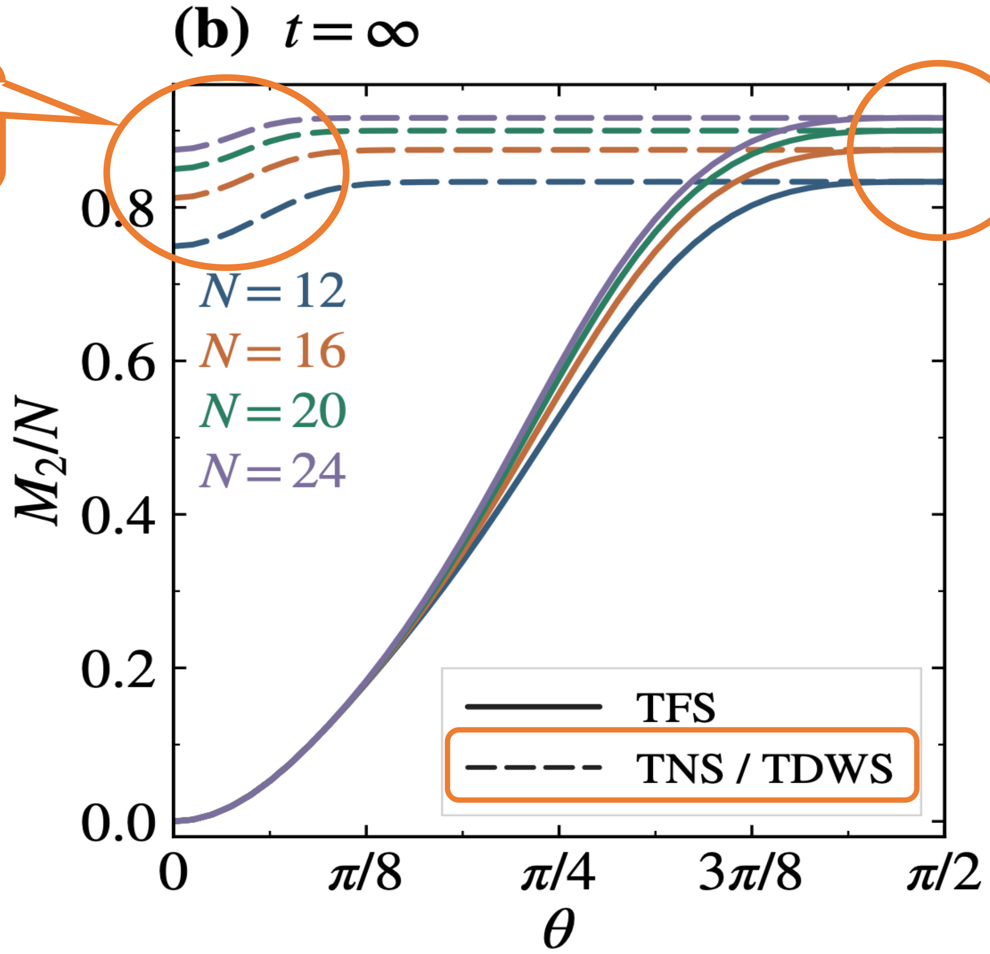
Late time nonstabilizerness

Late-time behavior of SRE:

$Q = N/2$

$|0011\cdots\rangle, |0101\cdots\rangle,$
$|0110\cdots\rangle, |1100\cdots\rangle$

Many states



$Q = N/2$

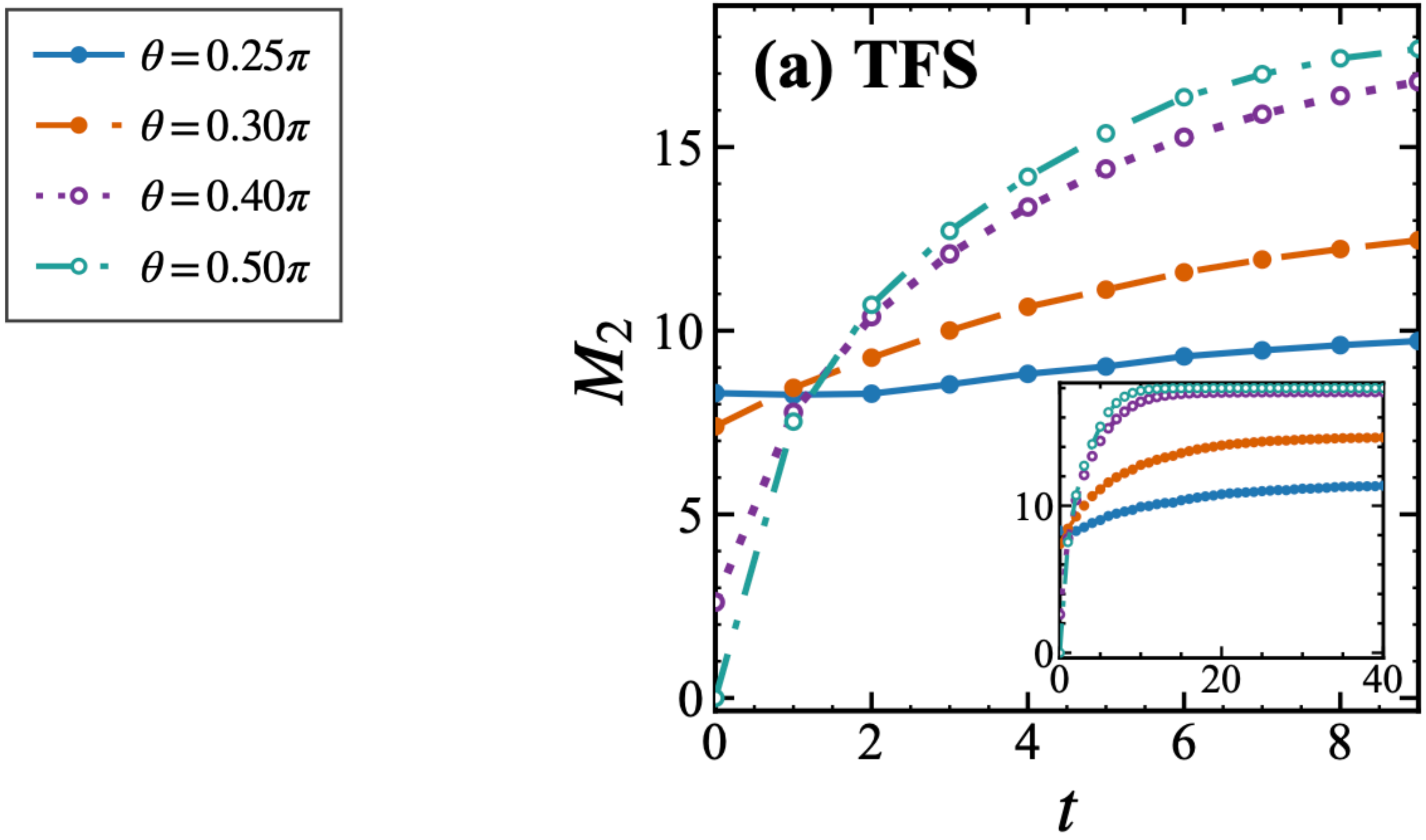
$|0011\cdots\rangle, |0101\cdots\rangle,$
$|0110\cdots\rangle, |1100\cdots\rangle$

Many states

Charge-conserving random circuit → **The initial charge distribution significantly affects the late-time SRE.**

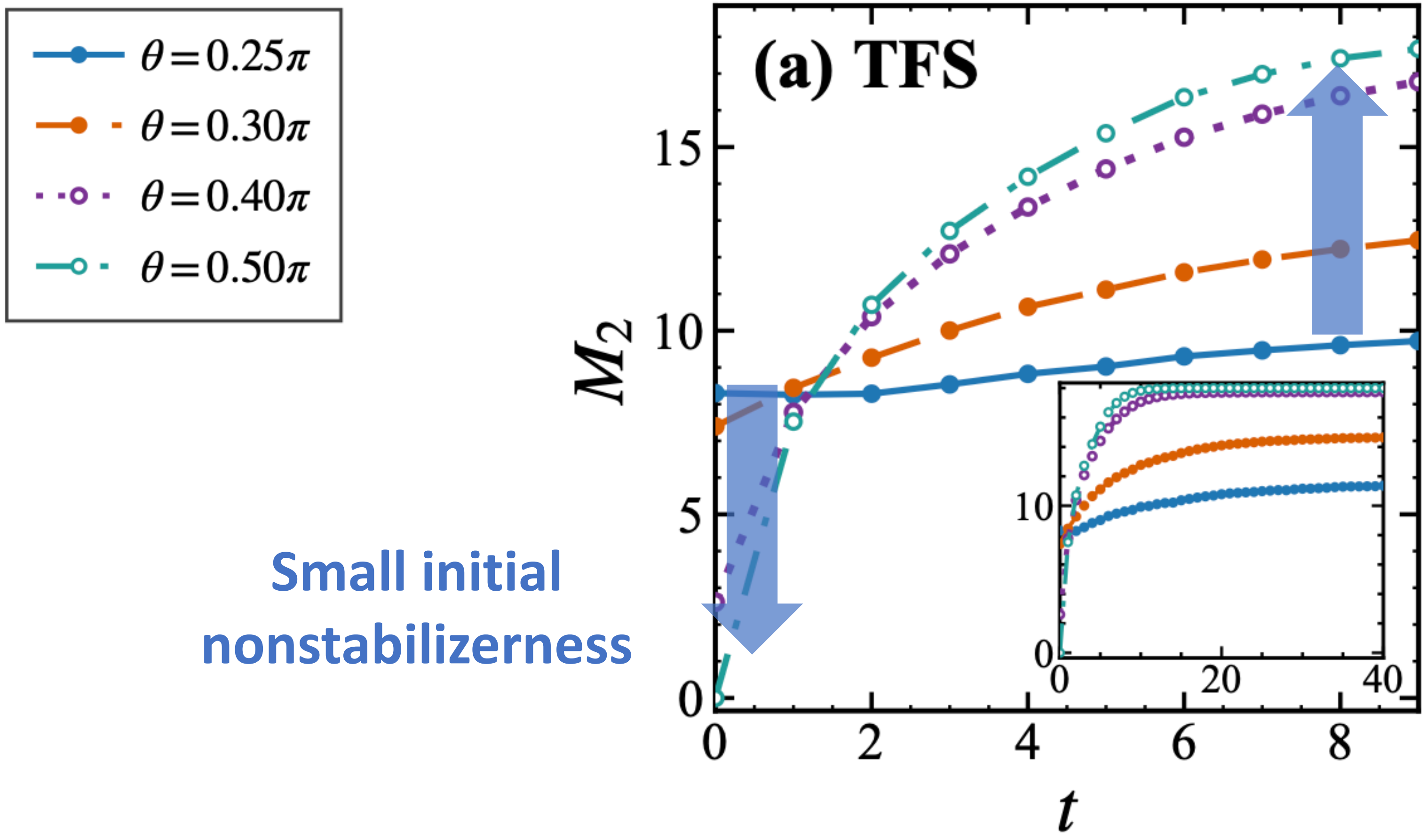
3. Nonstabilizerness Mpemba Effects

Time evolution of SRE:



3. Nonstabilizerness Mpemba Effects

Time evolution of SRE:



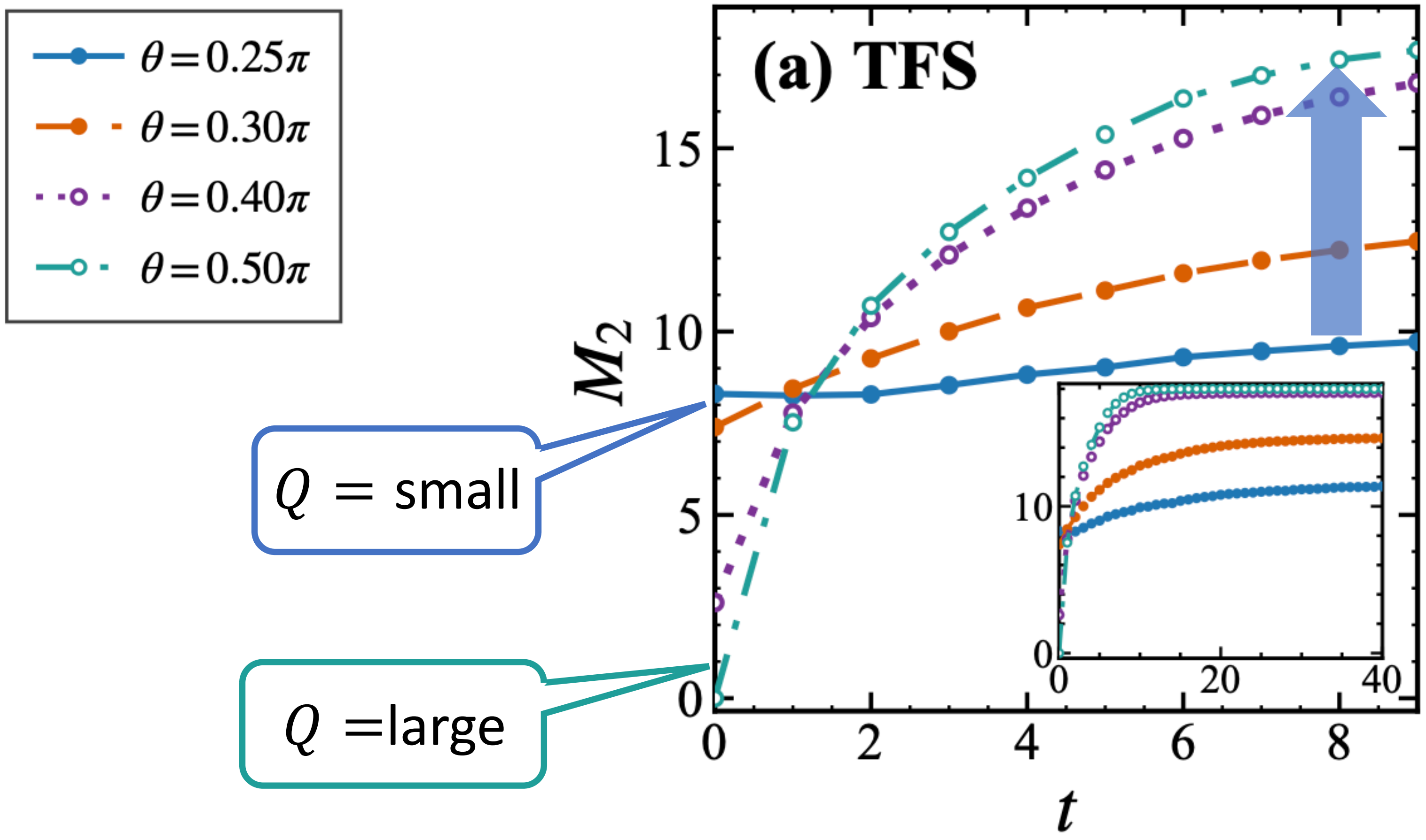
Faster generation of nonstabilizerness

Small initial nonstabilizerness

We uncover a nonstabilizerness Mpemba Effect!

3. Nonstabilizerness Mpemba Effects

Time evolution of SRE:



Faster generation of nonstabilizerness

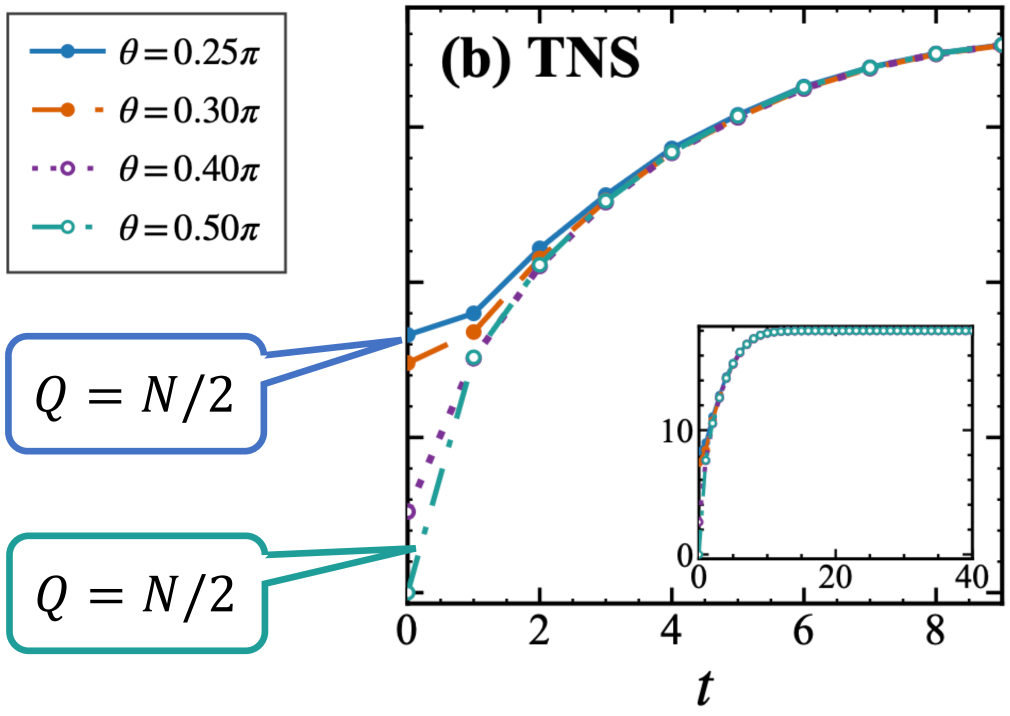
A similar behavior appears in the quantum Mpemba effect!

Large charge states tend to generate more nonstabilizerness

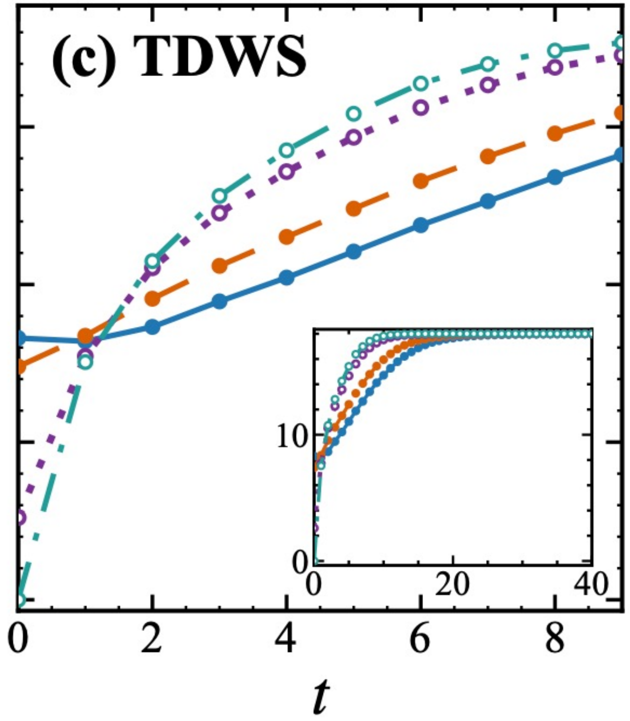
3. Nonstabilizerness Mpemba Effects

TNS $|\psi(\theta)\rangle_{\mathrm{TFS}} = e^{-i\frac{\theta}{2}\sum_{j=1}^{N} Y_j}|0101\cdots01\rangle$

TDWS $|\psi(\theta)\rangle_{\mathrm{TFS}} = e^{-i\frac{\theta}{2}\sum_{j=1}^{N} Y_j}|0\cdots01\cdots1\rangle$



No Mpemba Effect!



3. Nonstabilizerness Mpemba Effects

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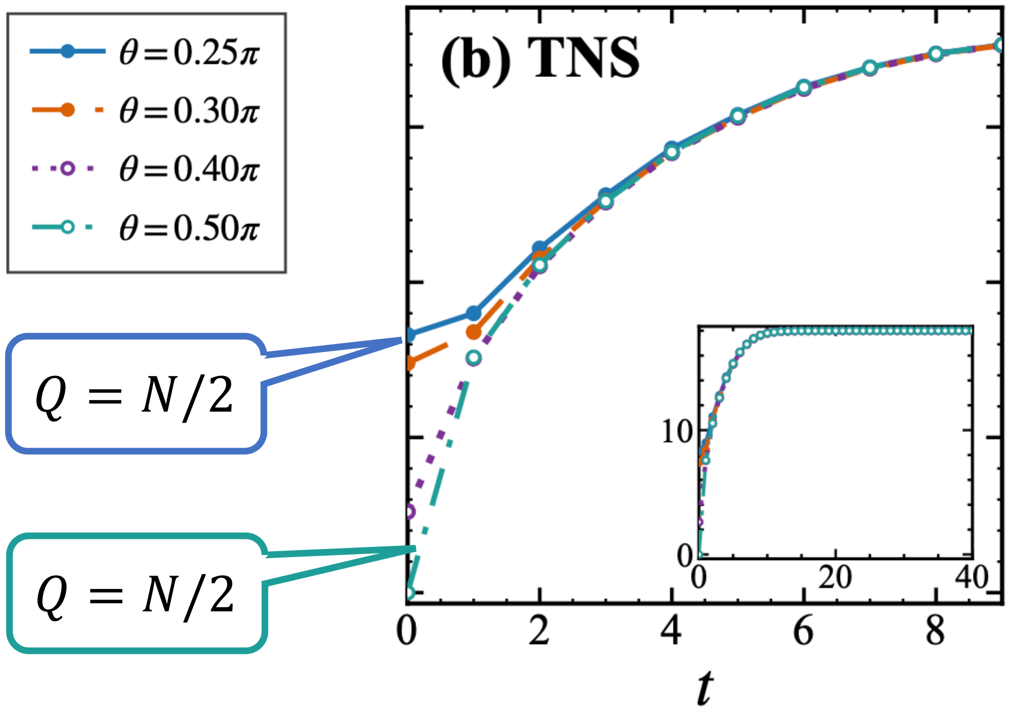
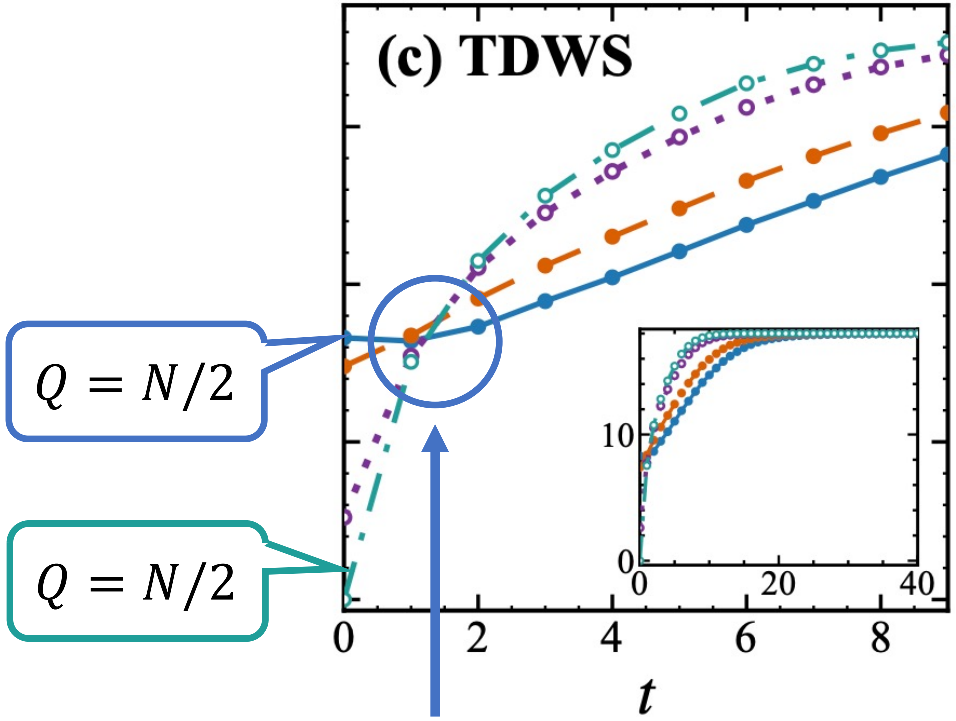


No Mpemba Effect!



Nonstabilizerness Mpemba Effect!

The spatial structure of the initial state also contributes to the Mpemba effect.

Outline

1. Introduction

2. Quantum Computing and Nonstabilizerness

3. Nonstabilizerness Mpemba Effects

4. Summary

4. Summary

- Mpemba Effect is an anomalous relaxation phenomenon discovered in physics.
  "Hot water can freeze faster than cold water."
- Nonstabilizerness is a resource for quantum advantage.
  Large nonstabilizerness is required to achieve quantum advantage.
- The authors uncovered a nonstabilizerness Mpemba effect.
  "A state with less initial nonstabilizerness can generate nonstabilizerness faster."

Take-home message

Physics $\xrightarrow{\text{conceptual}}$ **Quantum Computing**

"An interesting phenomenon discovered in physics may help us to generate a resource for quantum advantage."