HET Camp 2025@Osaka

# Entanglement asymmetry in Wess-Zumino-Witten model

Harunobu Fujimura

Collaborator: Soichiro Shimamori

2025/11/18 10:00~10:15

arXiv: 2509.05597v2

# 1. Introduction

Main topic of this talk:

**Quantum information theory** → Apply → **Symmetry breaking and its restoration**

**Take-home message:**

**A quantum-information perspective reveals a new aspect of symmetry breaking and its restoration.**

# 1. Introduction

**Conventional method** : order parameter $\langle \mathcal{O} \rangle$

$$\langle \mathcal{O} \rangle \begin{cases} = 0 : \text{symmetry presrevation} \\ \neq 0 : \text{symmetry breaking} \end{cases}$$

**New approach based on quantum information:**

**Entanglement asymmetry** = relative entropy [Ares-Murciano-Calabrese, 2022]

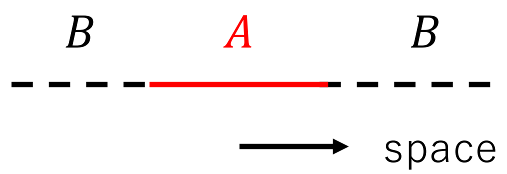
**Advantages:**
- Quantifies the degree of symmetry breaking
- Applicable to out-of-equilibrium systems.
- Enables discussion of the **Quantum Mpemba effect** (explained later)

# 1. Introduction

Let us consider the following out-of-equilibrium dynamics:

**Quench by a symmetric Hamiltonian**

Total system= $A \cup B$ , $A$: subsystem of interest




$|\psi_{AB}(0)\rangle$ : (explicit) Symmetry-broken initial state

$|\psi_{AB}(t)\rangle = e^{-iHt}|\psi_{AB}(0)\rangle$, $H$ : Hamiltonian with a symmetry

➡ **After some time, the symmetry is restored on subsystem $A$.**

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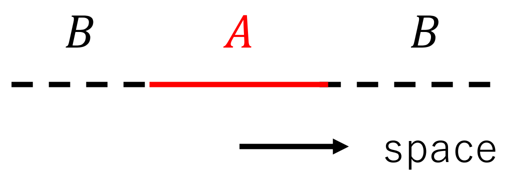


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**After some time, the symmetry is restored on subsystem $A$.**

The new approach uncovers the following phenomenon:

**Quantum Mpemba effect** ⋯The **more** symmetry is initially broken, the **faster** it is restored.

[Mpemba-Osborn, 1969] [Ares-Murciano-Calabrese, 2022]

**A counterintuitive phenomenon!** e.g., "Hot coffee can cool faster than warm coffee."

# 1. Introduction

## New approach based on quantum information [Ares-Murciano-Calabrese, 2022]

**Assumptions**

- Theory has a symmetry $G$.
- Decomposition of Hilbert space : $\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$

$$U_{tot}(g) = U_A(g) \otimes U_B(g), \quad g \in G$$

Symmetry operators are also decomposed.

### Entanglement Asymmetry (EA)

: A quantitative measure of symmetry breaking at the level of subsystem.

$$\Delta S_A \equiv \underbrace{\Delta S(\rho_A | \rho_{A,G})}_{\text{Relative entropy}} = \mathrm{Tr}_A\left[\rho_A\left(\log \rho_A - \log \rho_{A,G}\right)\right]$$

$\rho_A$: reduced density matric on subsystem $A$ (not symmetric in general)

$$\rho_{A,G} \equiv \underbrace{\int_G \mathrm{d}g}_{\text{Haar integral}} \, U_A(g)\rho_A U_A^\dagger(g)$$

: symmetrized density matrix

# 1. Introduction

## Important properties of EA

1. Defined for any states→valid even for out of equilibrium.

2. Non-negativity:

$$\begin{cases} \Delta S_A = 0 \iff [\rho_A, U_A(g)] = 0 & \text{(symmetry preservation)} \\ \Delta S_A > 0 \iff [\rho_A, U_A(g)] \neq 0 & \text{(symmetry breaking)} \end{cases}$$

[Kullbck-Leibler, 1951]

3. Quantifies the degree of symmetry breaking on a subsystem

$$0 < \Delta S_A < \Delta S'_A$$

Degree of symmetry breaking: Small Large

➡ EA serves a quantitative measure of symmetry breaking on subsystem

# 1. Introduction

From this formalism, many works have discussed the quantum Mpemba effect though symmetry restoration.

**Previous studies** (about quantum Mpemba effect)

- Quantum spin chains (1d, 2d, integrable system) [Murciano et al, 2023] and so on
- Experimental observations on quantum computers [Joshi et al, 2024] and so on
- (1+1)d CFT (but only $U(1)$) [Benini-Godet-Singh, 2024]

⇨ Most previous studies focus on Abelian symmetries such as $U(1)$ or $\mathbb{Z}_N$.

**The Quantum Mpemba effect for non-Abelian symmetries remains unknown (especially in QFT).**

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**The Quantum Mpemba effect for non-Abelian symmetries remains unknown (especially in QFT).**

**Our work**

- We investigate the EA for $SU(N)$ symmetry for general $N$ using Wess-Zumino-Witten model.
- We demonstrate the existence of a **quantum Mpemba effect for non-Abelian symmetry for the first time**.
- **We uncover a new type of quantum Mpemba effect** (see later)

# 2. Our work

# 2. Our work

Let's consider the $\widehat{su}(N)_k$ Wess-Zumino-Witten model (2d CFT)

Mermin-Wagner theorem prohibits spontaneous symmetry breaking. [Mermin-Wagner, 1966]

### Initial state

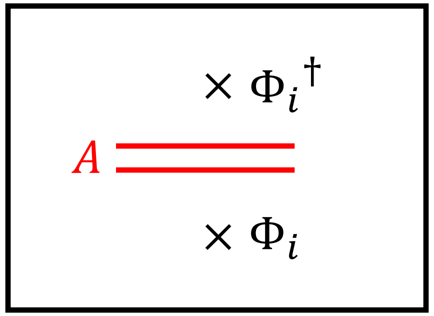
$$|\psi_{AB}(t=0)\rangle = \Phi_i(x_0, \tau_0)|0\rangle$$

$\Phi_i$: primary field in found. rep. $(i = 1, \ldots, N)$

**$SU(N)$ symmetry broken state (explicitly)**

Focus on Subsystem $A$

$$\rho_A = \mathrm{Tr}_B[|\psi_{AB}\rangle\langle\psi_{AB}|] =$$




Euclid time

space

Path integral rep.

### Quantity of interest: Rényi EA

$$\Delta S_A^{(n)} \equiv \frac{1}{1-n}\log\frac{\mathrm{Tr}_A[\rho_{A,G}^n]}{\mathrm{Tr}_A[\rho_A^n]}, \qquad \lim_{n\to 1}\Delta S_A^{(n)} = \Delta S_A$$

**By analyzing the time evolution of the Rényi EA, we can study the symmetry restoration dynamics on the subsystem.**

# 2. Our work

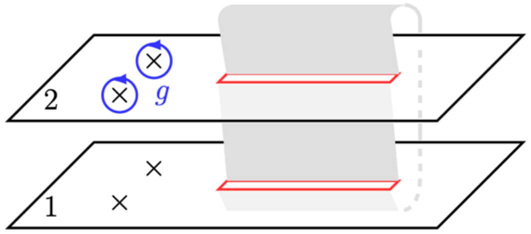
For simplicity, let us consider $\Delta S_A^{(2)}$ (the case $n = 2$).

## Flow of our analysis:

### Replica trick

[Benini et al, 2024]

$\mathrm{Tr}_A[\rho_{A,G}^2] \sim$

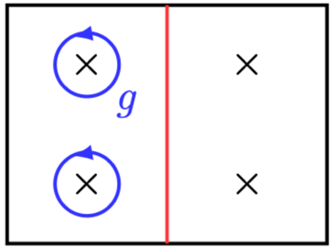



$\mathcal{R}_2$

### Conformal transf.

[He et al, 2024]

$\mathcal{R}_2 \rightarrow$




$\mathbb{R}^2$

### 4pt function

[Knizhnik et al, 1984]

$$\langle \Phi_i {\Phi_j}^\dagger \Phi_k {\Phi_\ell}^\dagger \rangle$$

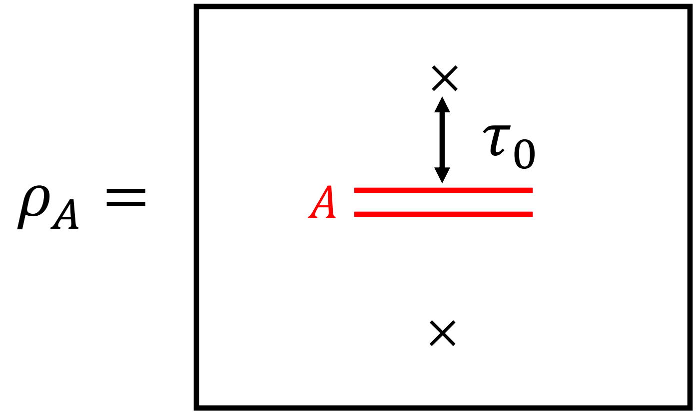
→Solvable analytically.

**By combining these methods, we analytically derived the Rényi EA in $\widehat{su}(N)_k$ WZW model.**

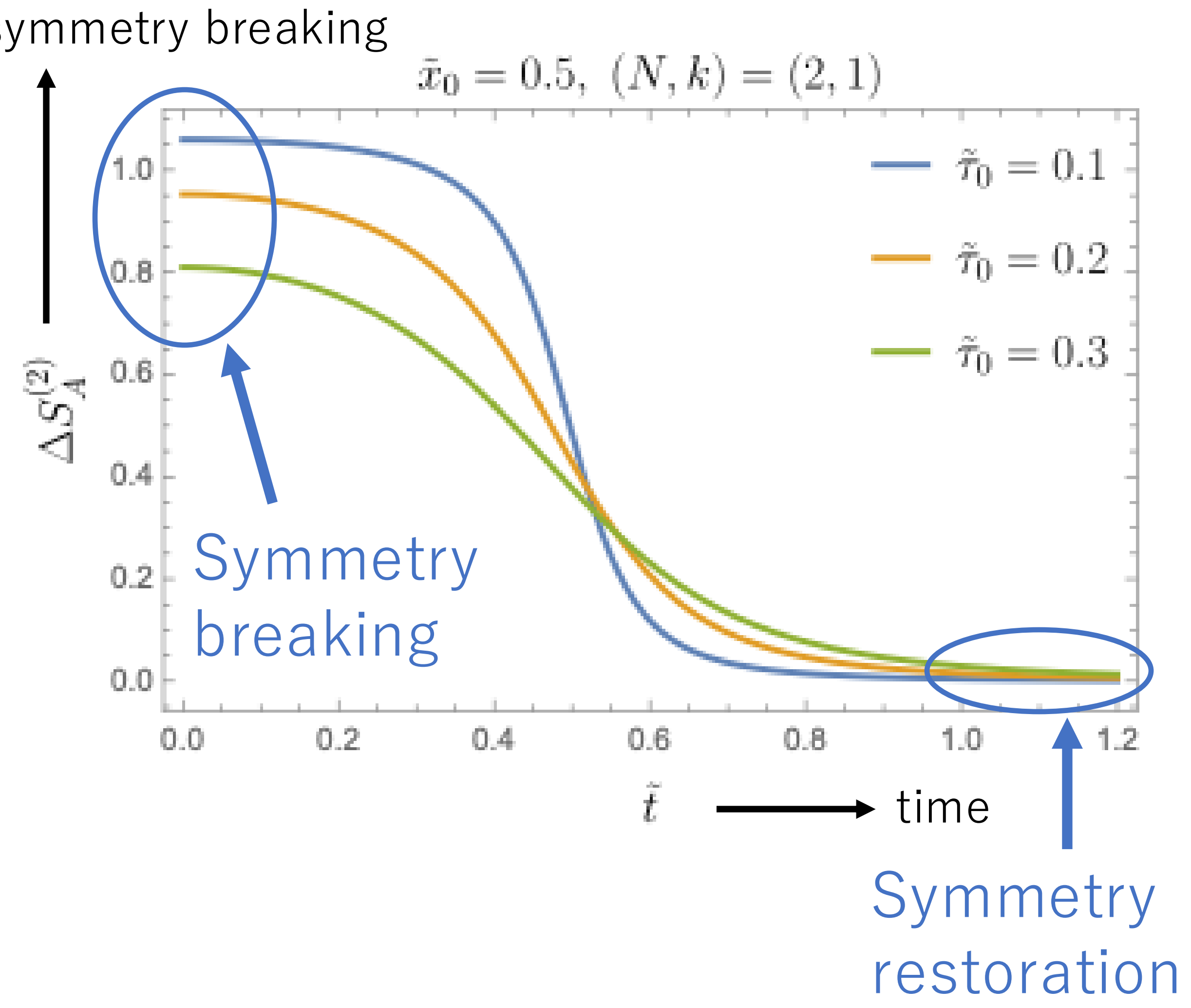
# 2. Our work

Let us examine the $SU(N)$ symmetry-restoration dynamics.

**Initial state**



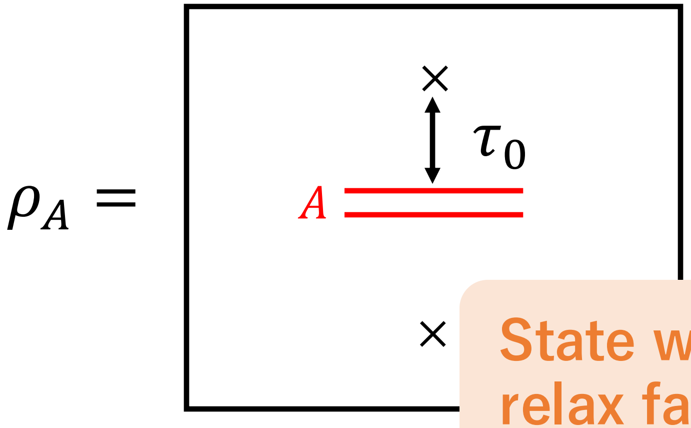

$\tau_0$: parameter of initial symmetry breaking



# 2. Our work

Let us examine the $SU(N)$ symmetry-restoration dynamics.

**Initial state**

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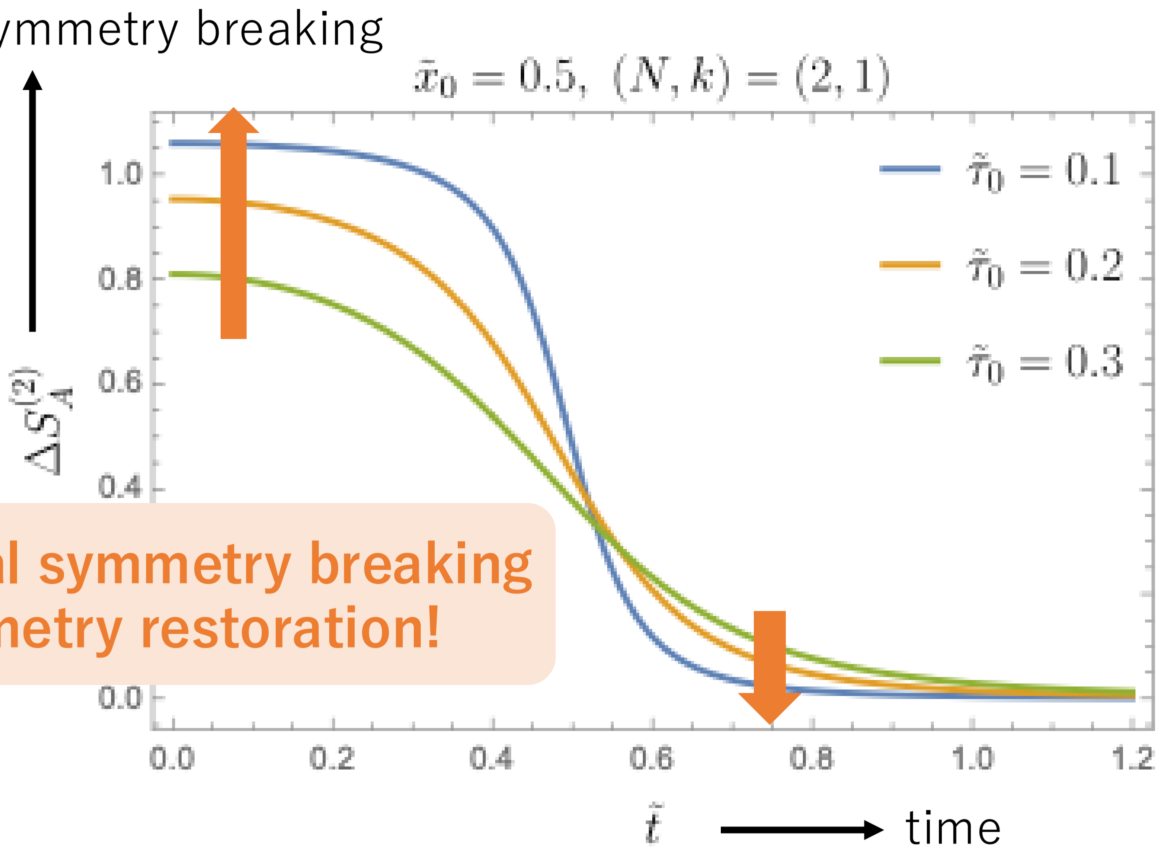
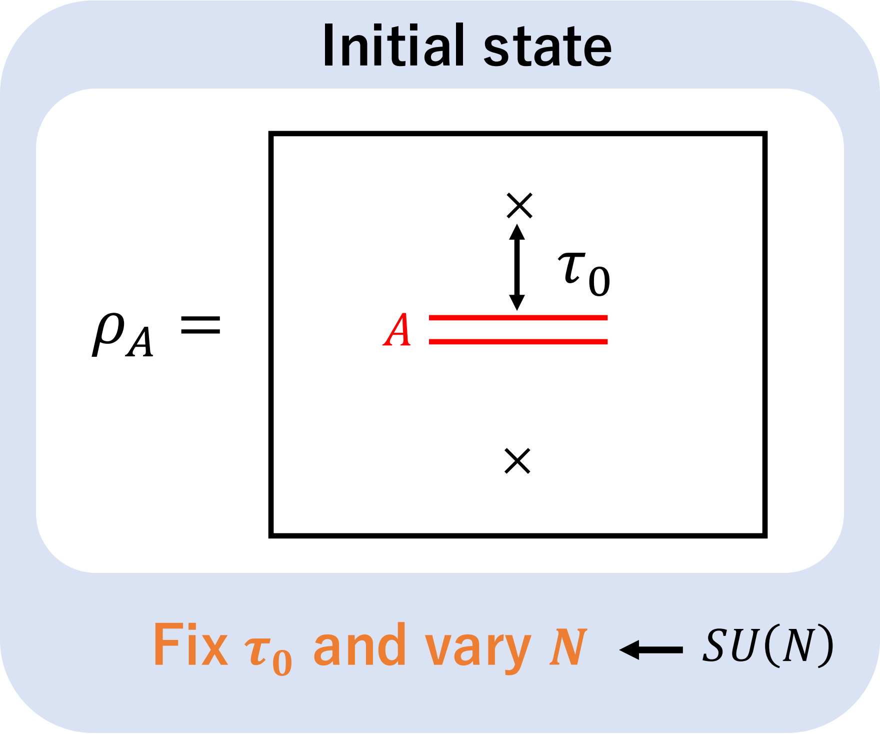


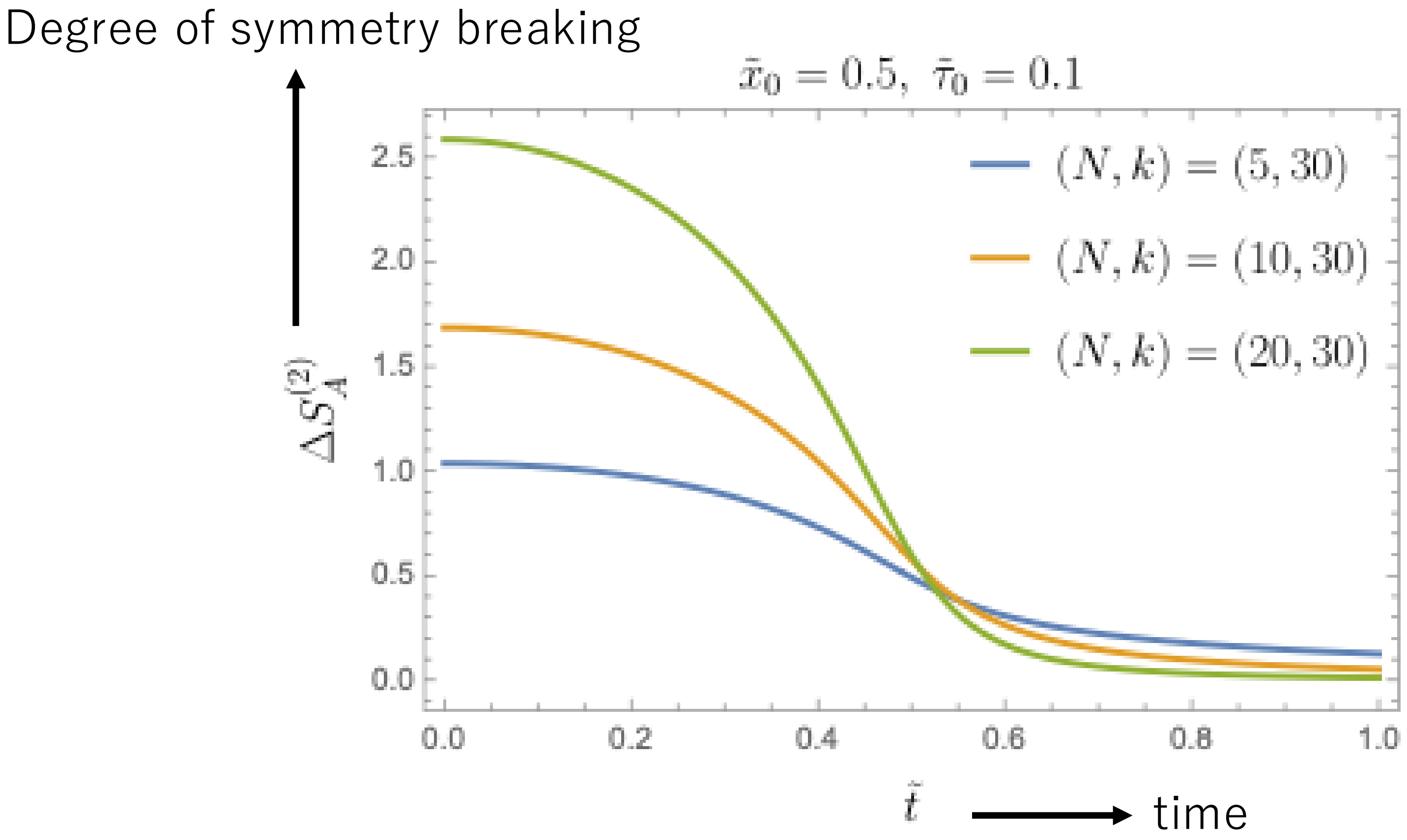

**State with stronger initial symmetry breaking relax faster toward symmetry restoration!**

**We analytically show the quantum Mpemba effect for non-Abelian symmetry.**

# 2. Our work

Let us consider the rank $N$ dependence.



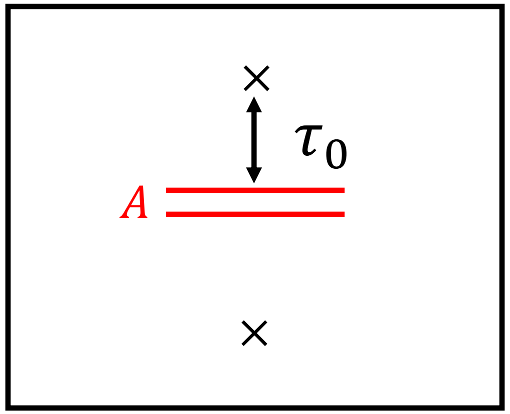



# 2. Our work

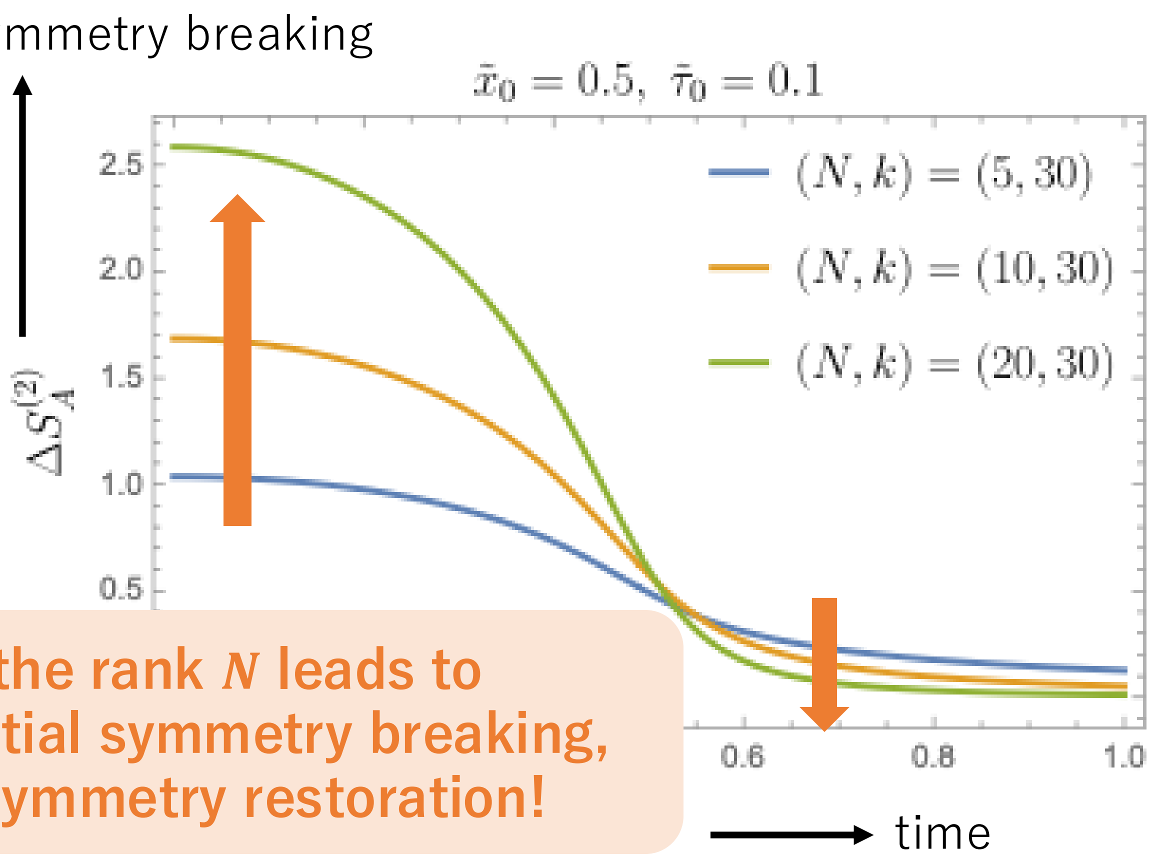
Let us consider the rank $N$ dependence.

**Initial state**

$\rho_A =$




**Fix $\tau_0$ and vary $N$** $\longleftarrow SU(N)$




**Increasing the rank $N$ leads to stronger initial symmetry breaking, but faster symmetry restoration!**

## We uncover a new type of quantum Mpemba effect!

# 3. Summary

We presented part of our results.

### Other contents (not showed here)

- Physical interpretation via the quasiparticle picture
- Level-$k$ dependence and associated quantum Mpemba effect.
- Another initial state： $|\psi_{AB}(t=0)\rangle = J^a|0\rangle$, $J^a$: WZW current of adjoint rep.

arXiv: 2509.05597v2

### Summary of this talk

- Recently, symmetry has been actively studied from a quantum information perspective
  →Quantum Mpemba effect
- Previous works have mainly focused on Abelian symmetries:
  the quantum Mpemba effect for non-Abelian symmetries were not understood.
- In this work, **we showed the quantum Mpemba effect for non-Abelian symmetry for the first time**.
- Furthermore, **we uncovered a new type of quantum Mpemba effect**.

# Appendix

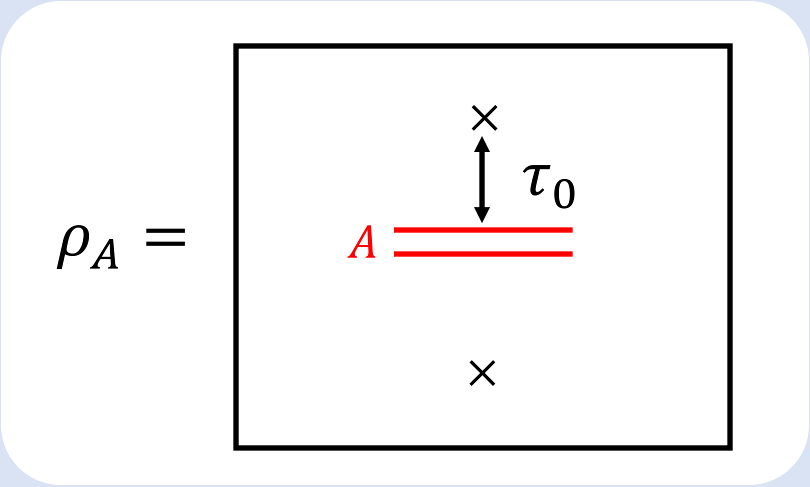
# Appendix: future directions

- Extension to $\widehat{so}(N)_k$ and $\widehat{sp}(N)_k$ WZW model→analysis is completely parallel.
- In this work, we consider fundamental and adjoint reps
  → extension to other representation.
- Quantum Mpemba effect in finite temperature systems.
- Microscopic mechanism of the quantum Mpemba effect.

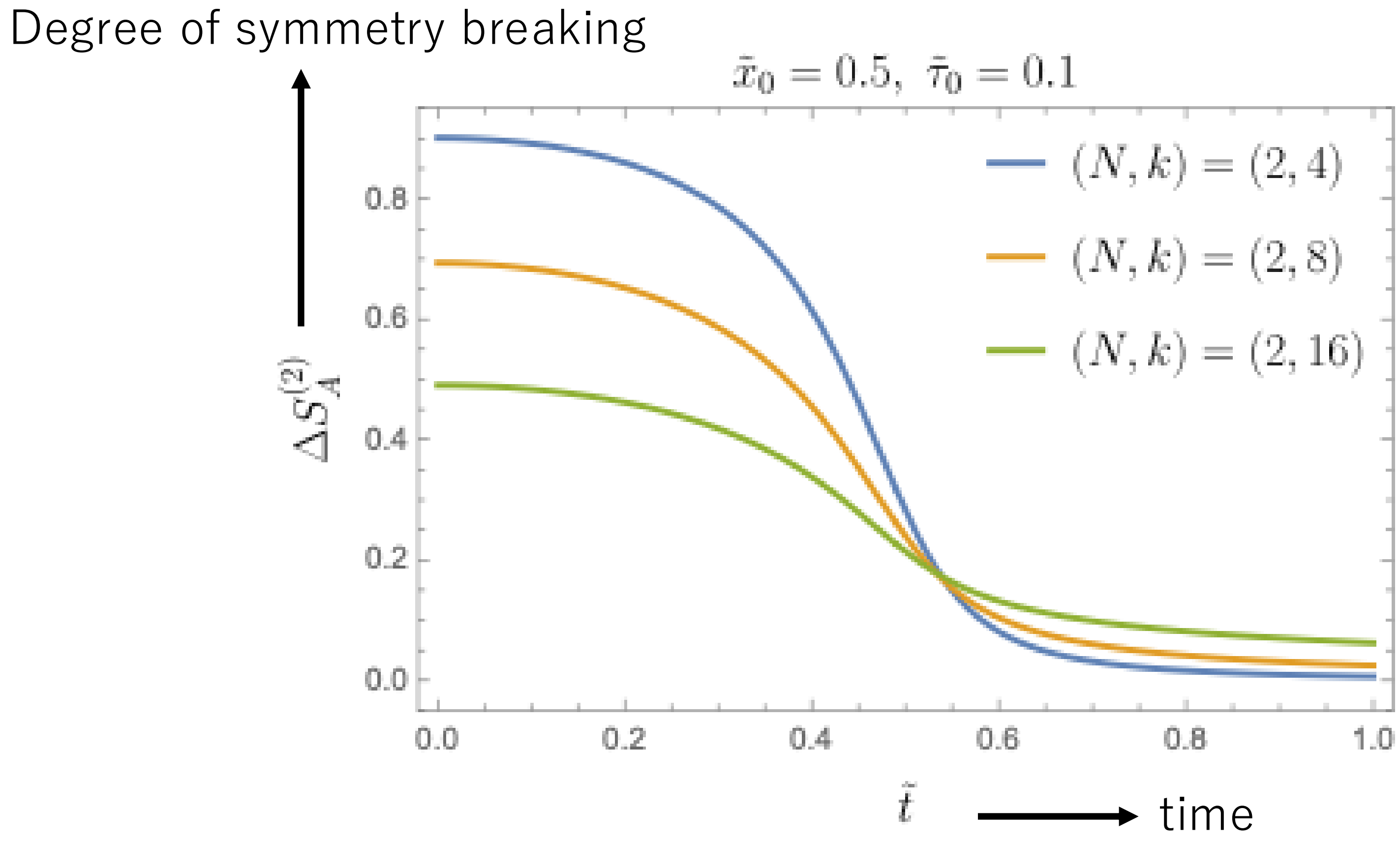
# Appendix: level $k$ dependence

Let us consider the level-$k$ dependence with fixed rank $N$.

**Initial state**




**Fix $\tau_0$ and vary level $k$**



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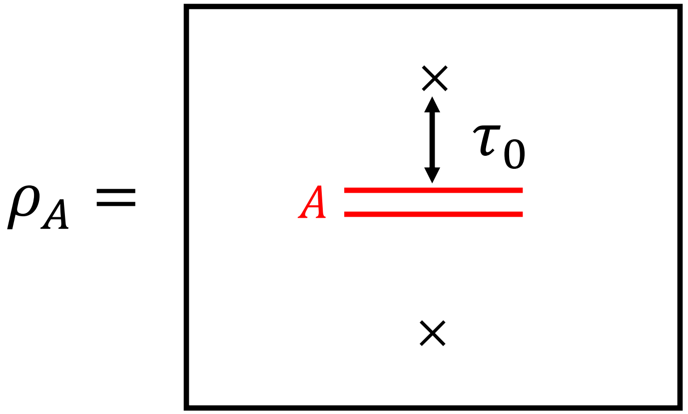


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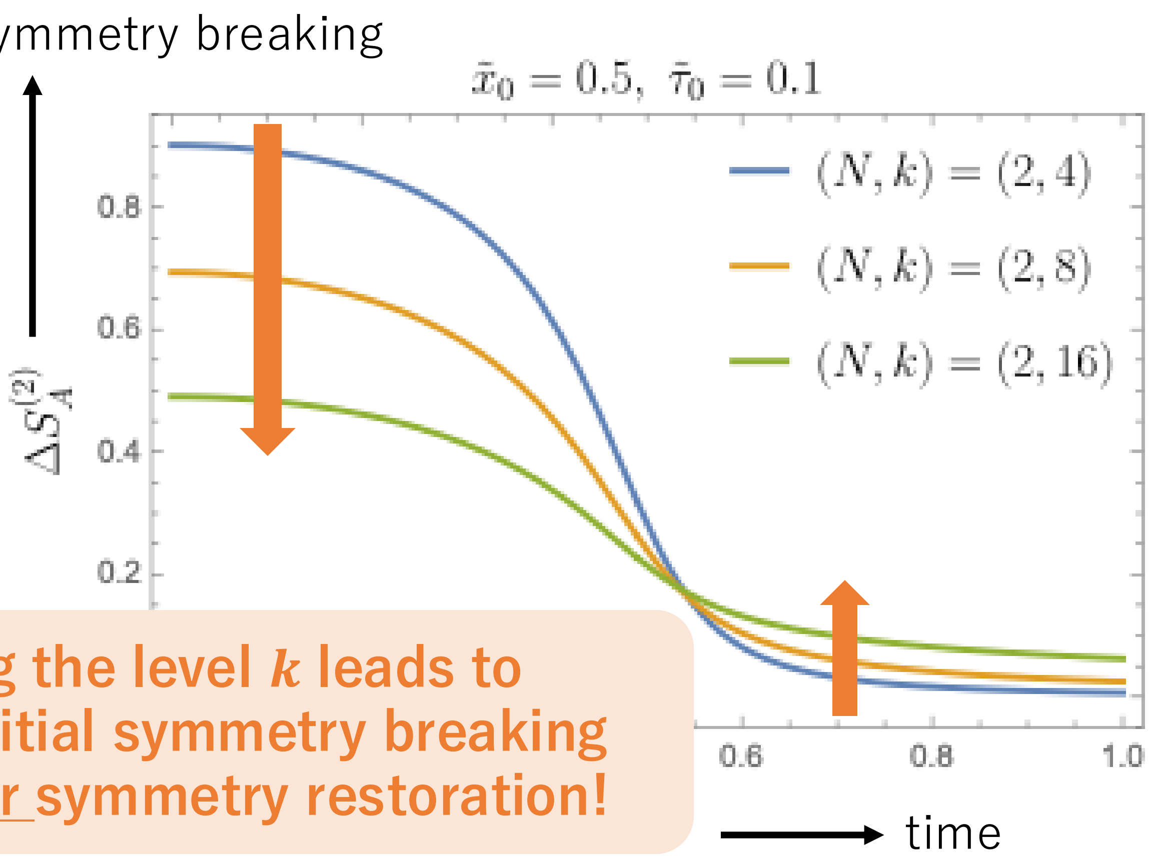



**Increasing the level $k$ leads to <u>weaker</u> initial symmetry breaking but <u>slower</u> symmetry restoration!**

**This is also a new type of quantum Mpemba effect!**

# Appendix: other initial state

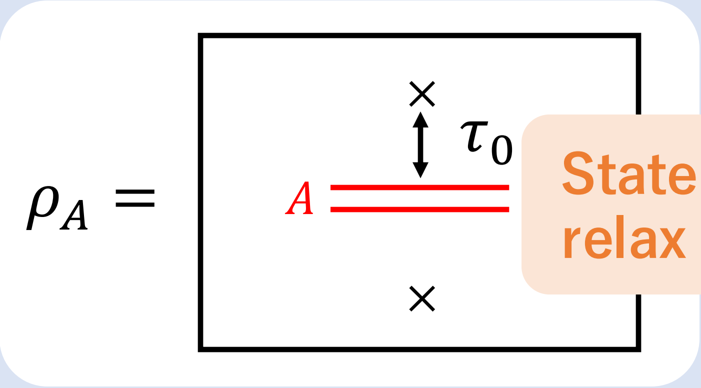
**Vary $\tau_0$, with $(N, k)$ fixed.**

**Initial state**

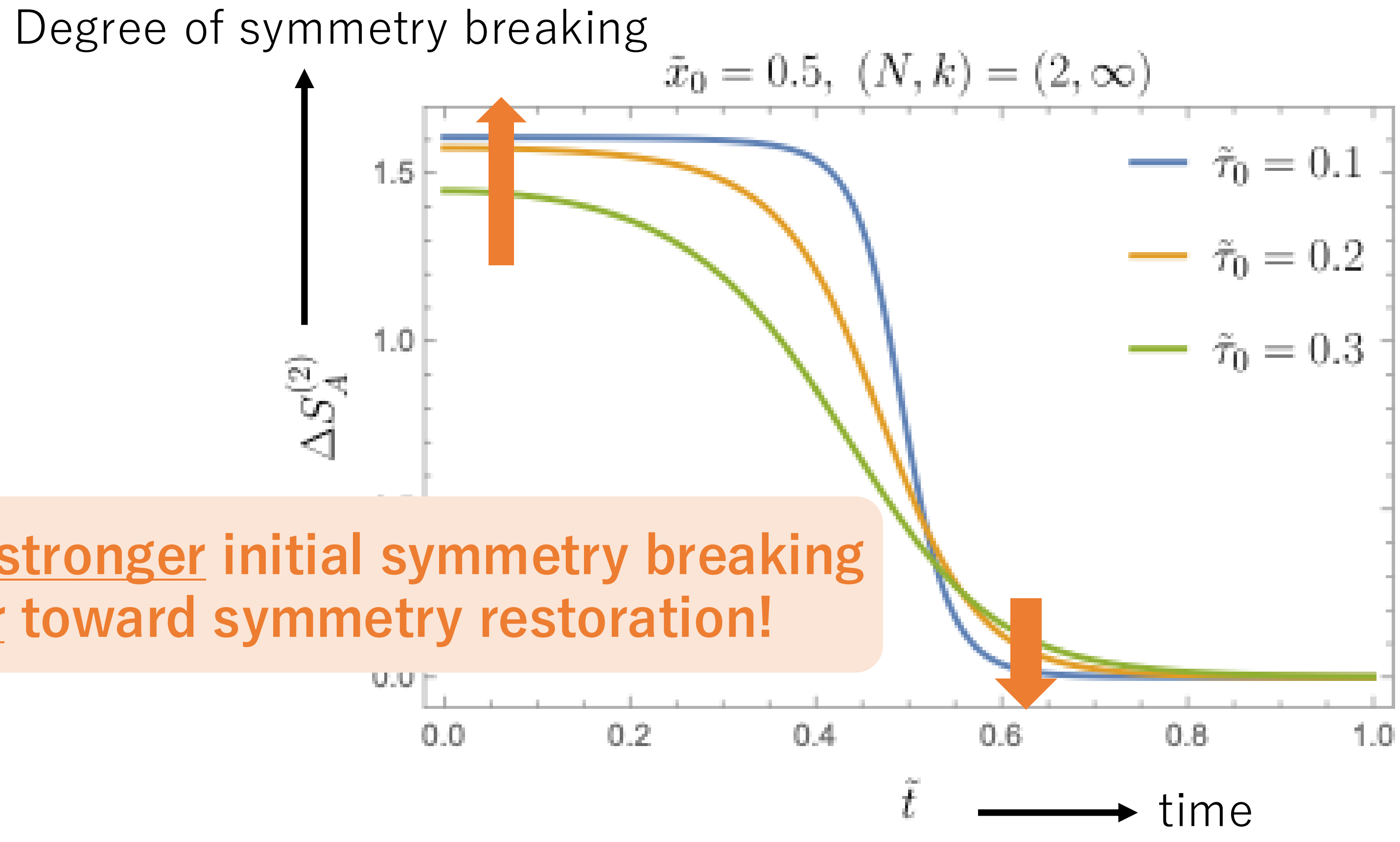
$$|\psi_{AB}(t=0)\rangle = J^a(x_0, \tau_0)|0\rangle$$

$J^a$ : WZW current in adjoint rep.

$a = 1, \dots, N^2 - 1$




Take limit $k \to \infty$ for simplicity




**State with stronger initial symmetry breaking relax faster toward symmetry restoration!**

**We also demonstrate the quantum Mpemba effect in this case**

# Appendix: Other initial state

**Vary $N$ with $\tau_0$ fixed.**

### Initial state

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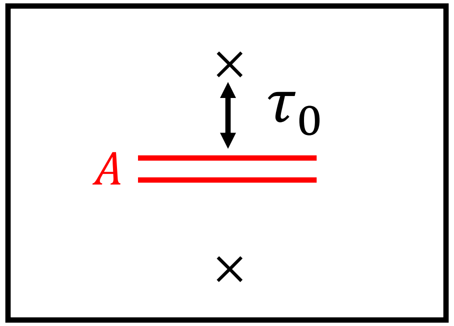
$\rho_A =$ 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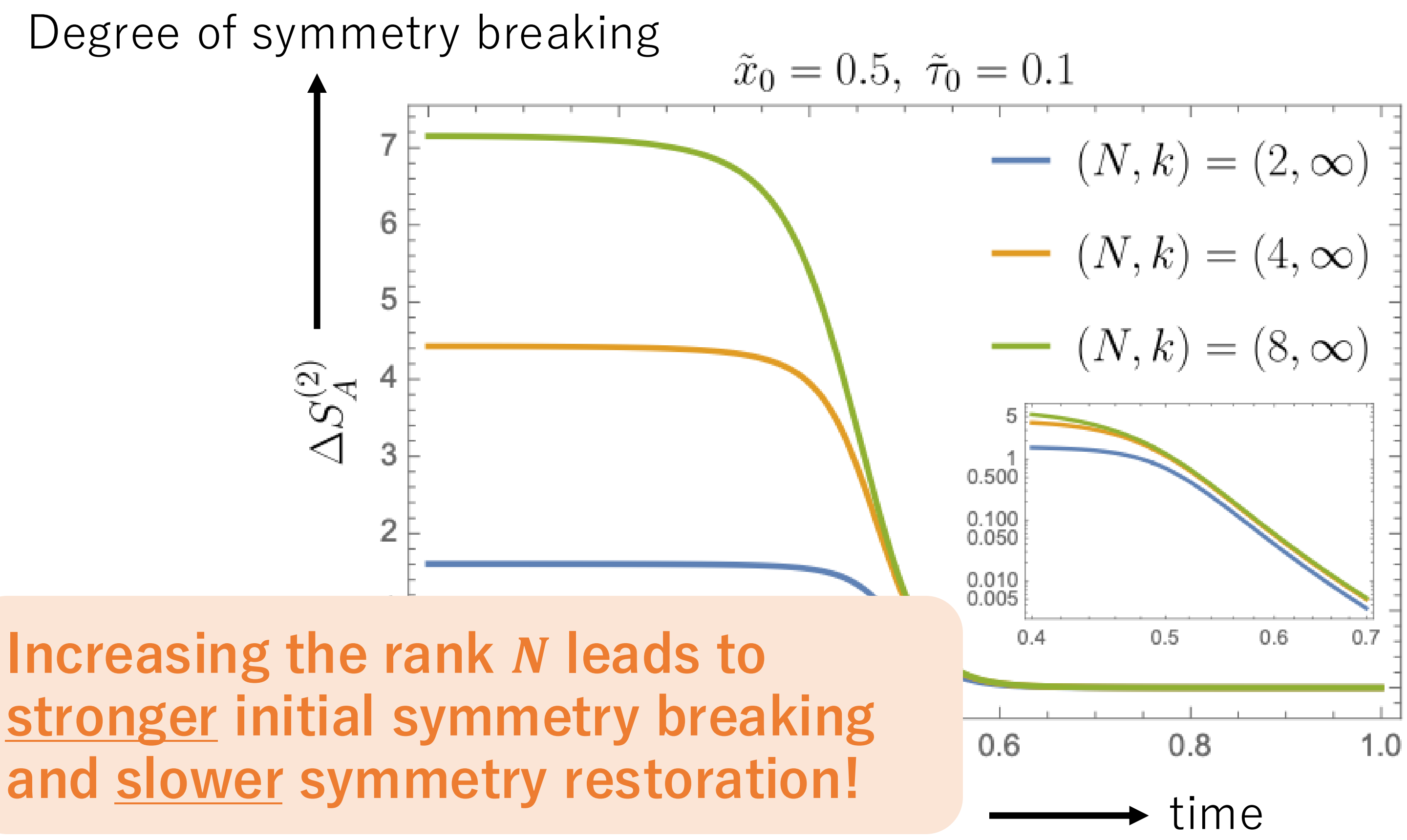



**Increasing the rank $N$ leads to <u>stronger</u> initial symmetry breaking and <u>slower</u> symmetry restoration!**

➡ **There is no new type of quantum Mpemba effect in this case**

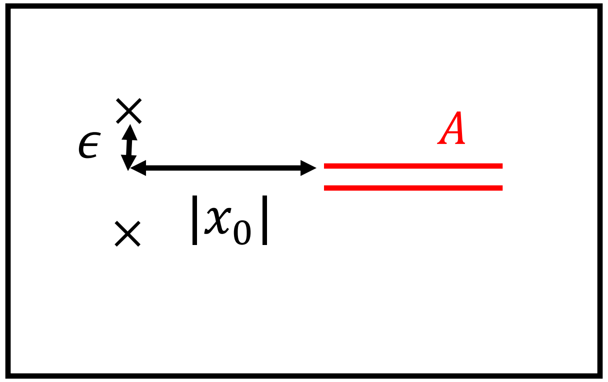
# Appendix: quasiparticle picture

**Initial state**

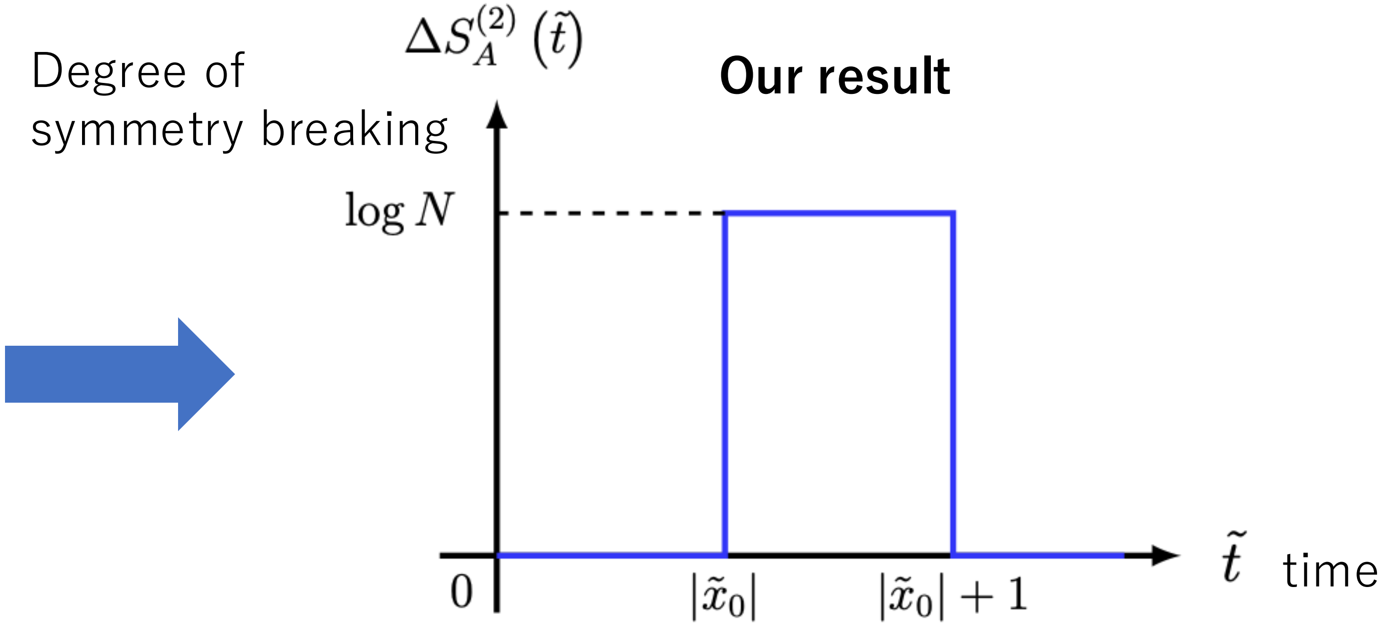
$$|\psi_{AB}(t=0)\rangle = \Phi_i(x_0,\tau_0)|0\rangle$$

$\Phi_i$: primary field in found. rep.
$(i = 1, \dots, N)$

Take limit $\tau_0 \to 0$

$\rho_A =$ 


**Our result**



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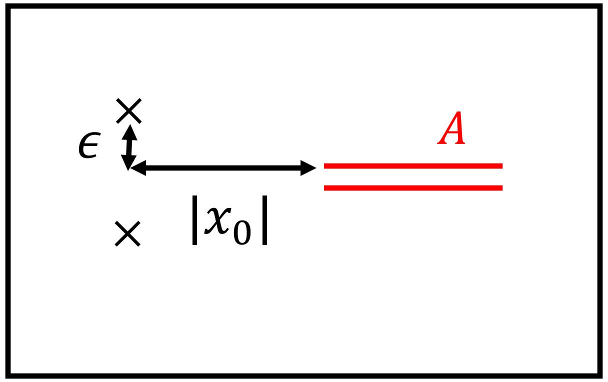
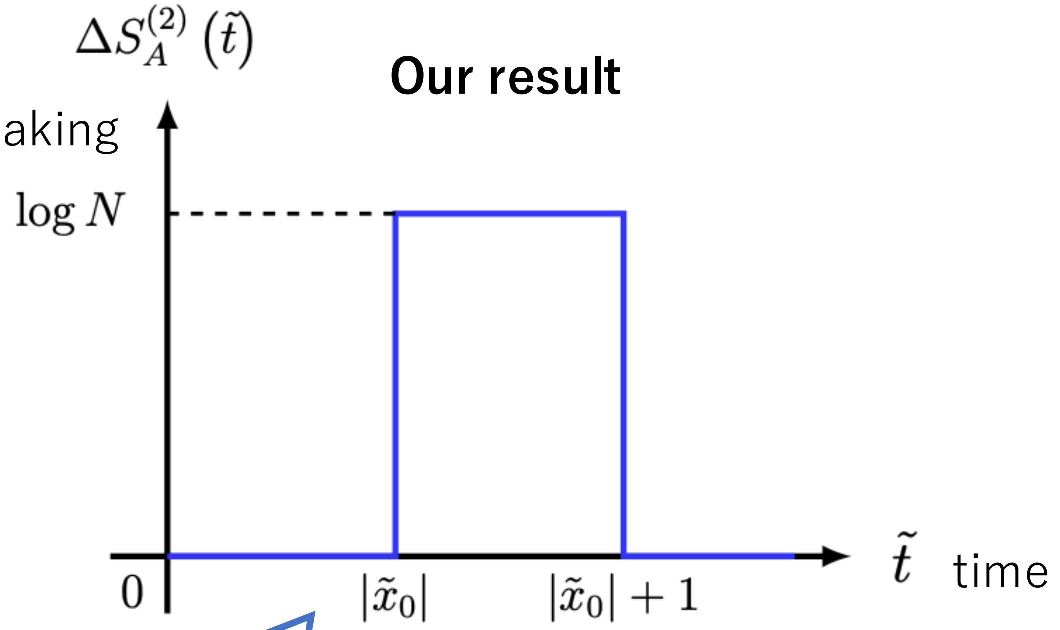
$\rho_A =$ 


Degree of symmetry breaking

**Our result**




**Physical interpretation**