Quantum entanglement for detecting non-invertible symmetry breaking

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Based on ongoing work with Soichiro Shimamori

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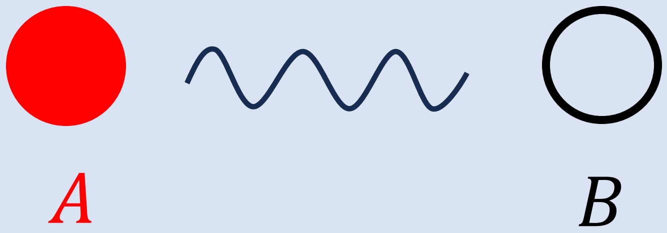
Introduction

Today's two topics;

Quantum Entanglement

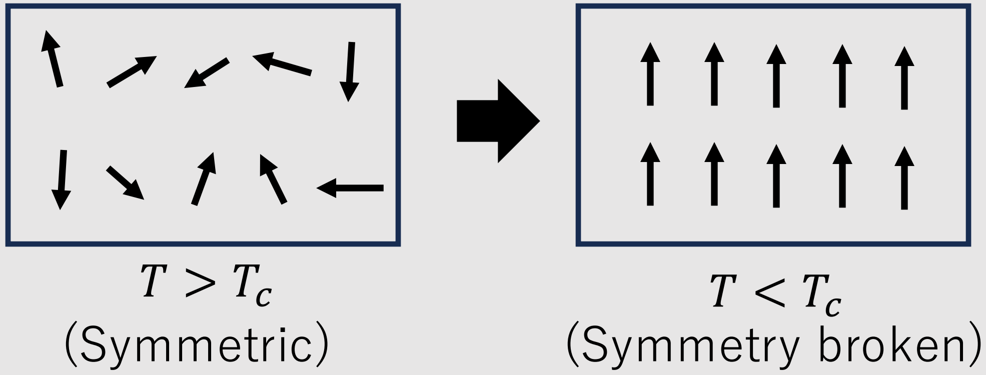
=Quantum correlations that has no counterpart in classical theory



Applications
- quantum computation
- quantum teleportation
- Holography , etc

Apply

Symmetry breaking



- liquid → solid transition
- Ferromagnetic of spin chain
- Chiral symmetry breaking in QCD , etc

I will show how entanglement can be a powerful tool for studying symmetry breaking.

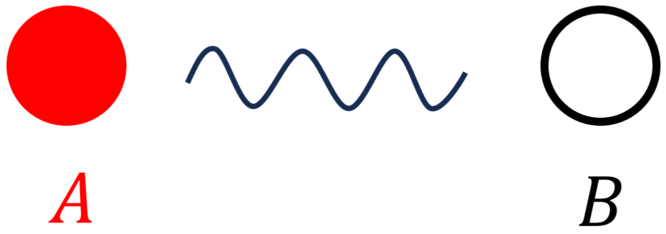
Introduction

How to quantify quantum entanglement?

State of total system: $|\psi_{AB}\rangle$

Density matrix ： $\rho_{\mathrm{AB}} = |\psi_{AB}\rangle\langle\psi_{AB}|$

Reduced density matrix： $\rho_A = \mathrm{Tr}_B[\rho_{\mathrm{AB}}]$



Entanglement Entropy(EE) :

$$S(\rho_A) \equiv -\mathrm{Tr}_A[\rho_A \log \rho_A]$$

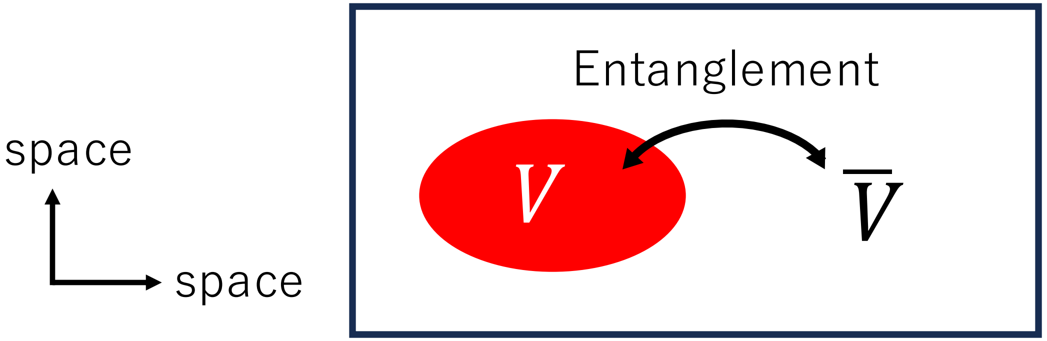
EE represent how much system A and B are quantum entangled.

Introduction

Quantum entanglement for QFT;

Field $\phi(x)$ → There are degrees of freedom on each special point.

Subsystem A → region V
Subsystem B → region $\overline{V}$ = complementary region of V



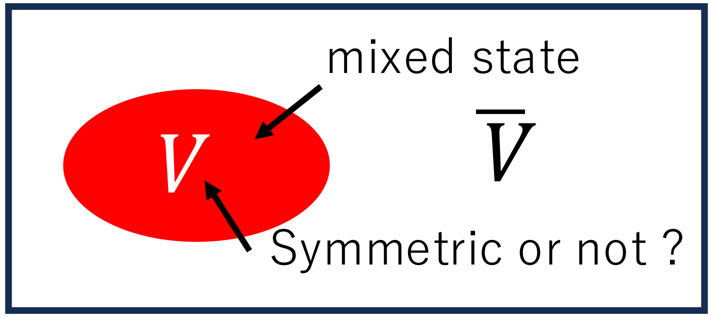
Entanglement Entropy for QFT :

$$S(\rho_V) \equiv -\mathrm{Tr}_V[\rho_V \log \rho_V]$$

ρ_V : The density matrix on region V

Introduction

How do we consider the symmetry breaking at the level of subsystem V(=mixed state)?



Conventional method using order parameter

・Typically, we assume a pure state $|\Psi\rangle$ (often the vacuum state).

Entanglement Asymmetry(EA) [Filiberto Ares, et al, 2022]

・EA is defined by entanglement entropy.
・ EA quantify <u>how much </u>the symmetry is breaking at the level of subsystem.
・It is valid for general mixed states.

Outline

Outline

1. Introduction
2. Entanglement Asymmetry
3. Application to QFT and non-invertible symmetry (Ongoing work)
4. Summary and future direction

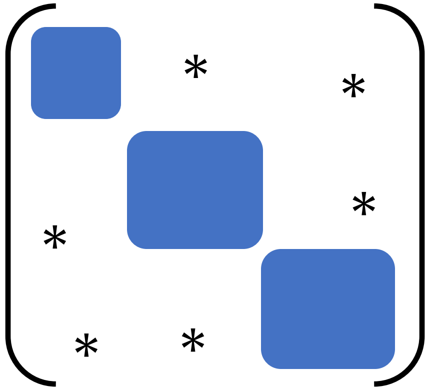
2. Entanglement Asymmetry

Let' consider a continuum symmetry whose charge is $Q = Q_A + Q_B$

where, $Q_{A/B}$ is charge that act on subsystem A/B.

$[\rho_A, Q_A] = 0$ or $\neq 0$: symmetric or not on A

$\rho_A =$

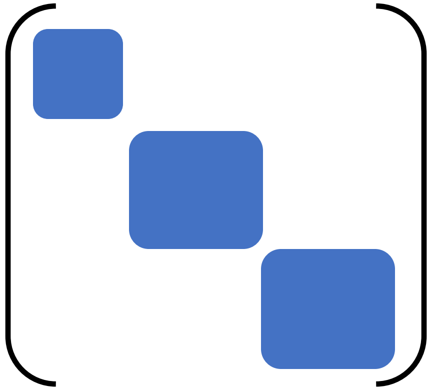


(Remove the off-diagonal elements)

projection

$$\rho_{A,Q} \equiv \sum_{q_A} \Pi_{q_A} \rho_A \Pi_{q_A}$$

$\rho_{A,Q} =$



Π_{q_A} : Projector onto the eigenspace of Q_A with charge q_A

$\rho_A = \rho_{A,Q}$ ⟺ Symmetric on subsystem A

$\rho_A \neq \rho_{A,Q}$ ⟺ Symmetry breaking on subsystem A

2. Entanglement Asymmetry

Entanglement Asymmetry (EA) [Filiberto Ares, et al, 2022]

$$\Delta S_A \equiv S(\rho_A | \rho_{A,Q}) \equiv \mathrm{Tr}[\rho_A(\log \rho_A - \log \rho_{A,Q})]$$

$$= S(\rho_{A,Q}) - S(\rho_A)$$

: Relative entropy (=distance between ρ_A and $\rho_{A,Q}$)

Important property [S. Kullback, et al 1951]

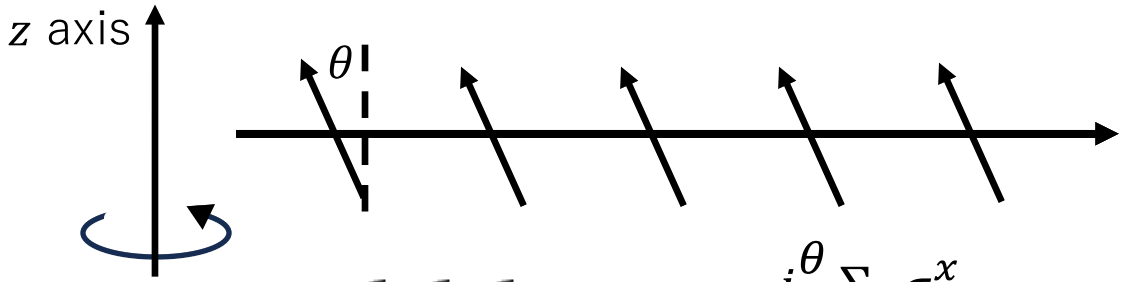
$\Delta S_A = 0 \Leftrightarrow \rho_A = \rho_{A,Q} \; ([\rho_A, Q_A] = 0)$ Symmetric on subsystem A

$\Delta S_A > 0 \Leftrightarrow \rho_A \neq \rho_{A,Q} \; ([\rho_A, Q_A] \neq 0)$ Symmetry breaking on subsystem A

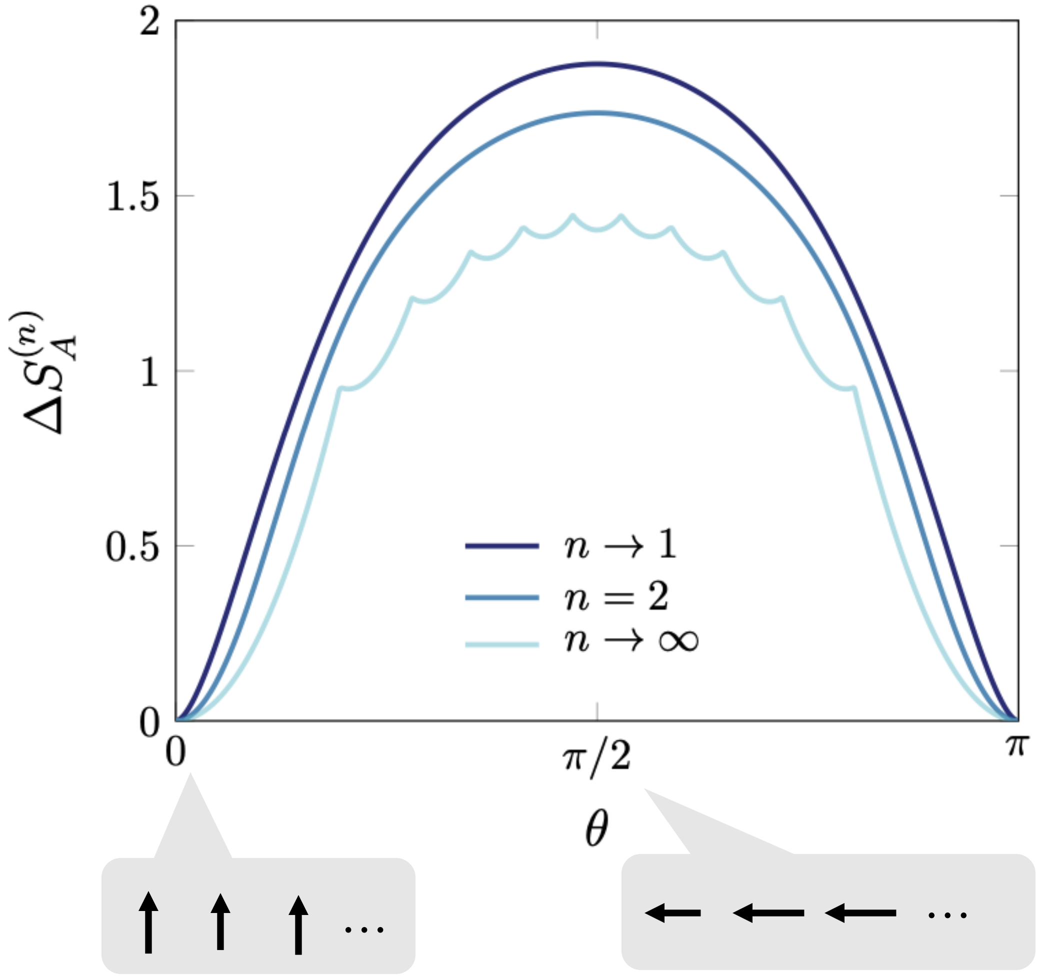
EA represents how much the symmetry is breaking on subsystem A

Example : Spin 1/2 chain

[Filiberto Ares, et al, 2022]



$$|\theta; \nearrow\nearrow\nearrow \cdots\rangle = e^{-i\frac{\theta}{2}\sum_j \sigma_j^x} | \uparrow\uparrow\uparrow \cdots \rangle$$



Consider the rotational symmetry around z axis

$$\left(Q = \frac{1}{2}\sum_j \sigma_j^z \right)$$

Projector : $\Pi_{q_A} = \int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} e^{-i\alpha q_A} e^{i\alpha Q_A}$

Rényi EA : $\Delta S_A^{(n)} = \frac{1}{1-n} \log\left(\frac{\mathrm{Tr}[\rho_{A,Q}^n]}{\mathrm{Tr}[\rho_A^n]} \right)$

2. Entanglement Asymmetry

Let's see the symmetry restoration

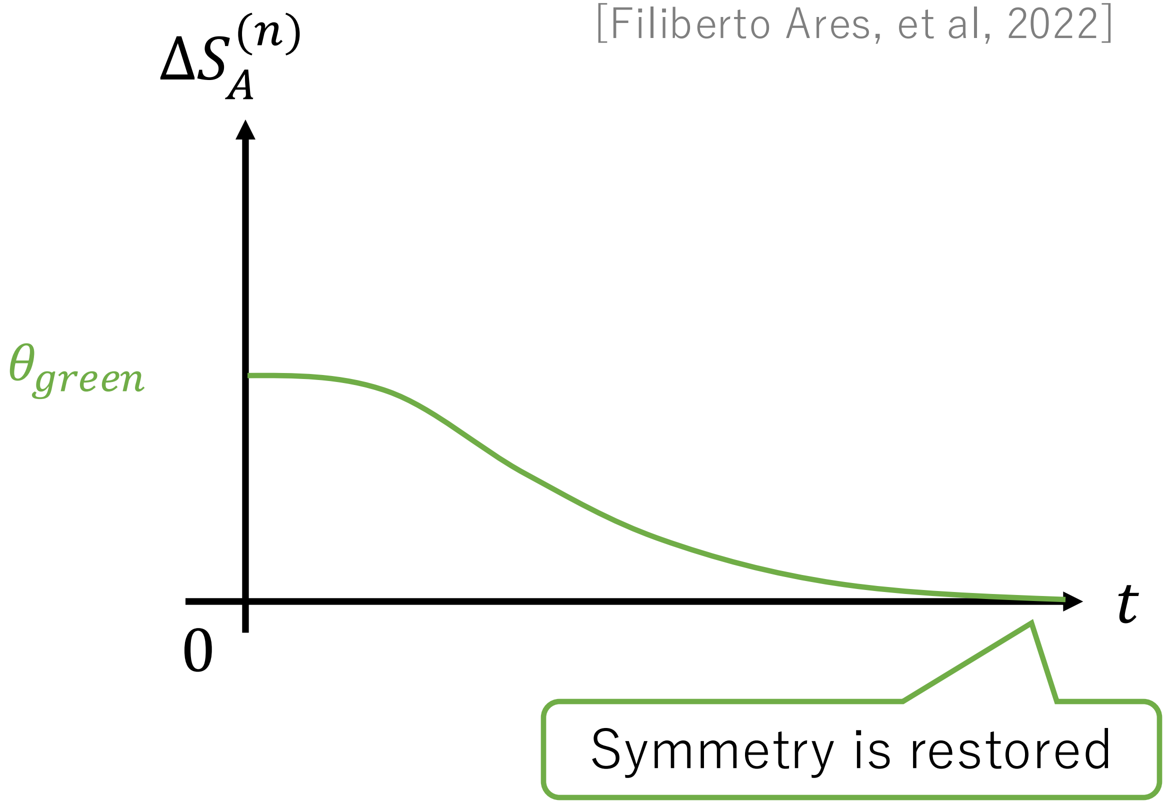
Quench of XX Hamiltonian

$$H = -\frac{1}{4}\sum_j \left(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y\right)$$

$$([H, Q] = 0)$$

$$|\Psi(t)\rangle = e^{iHt}|\theta; \nearrow\nearrow\nearrow \cdots\rangle$$

[Filiberto Ares, et al, 2022]



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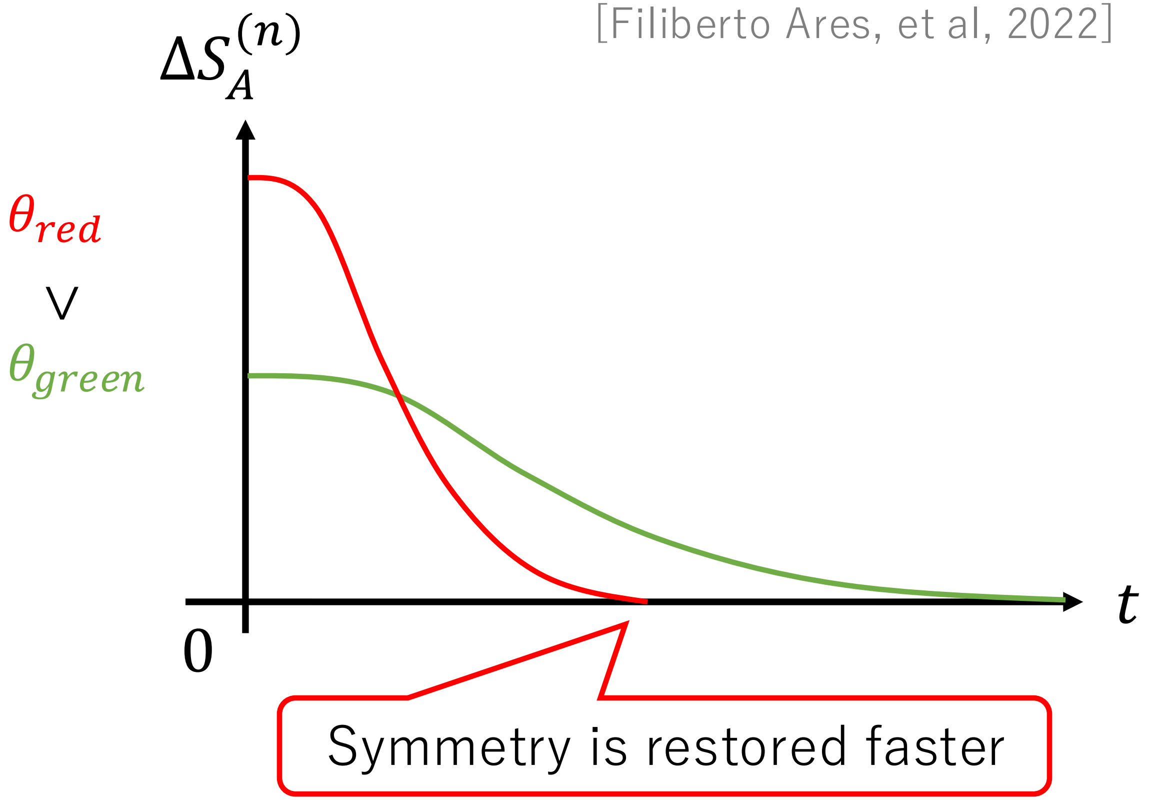

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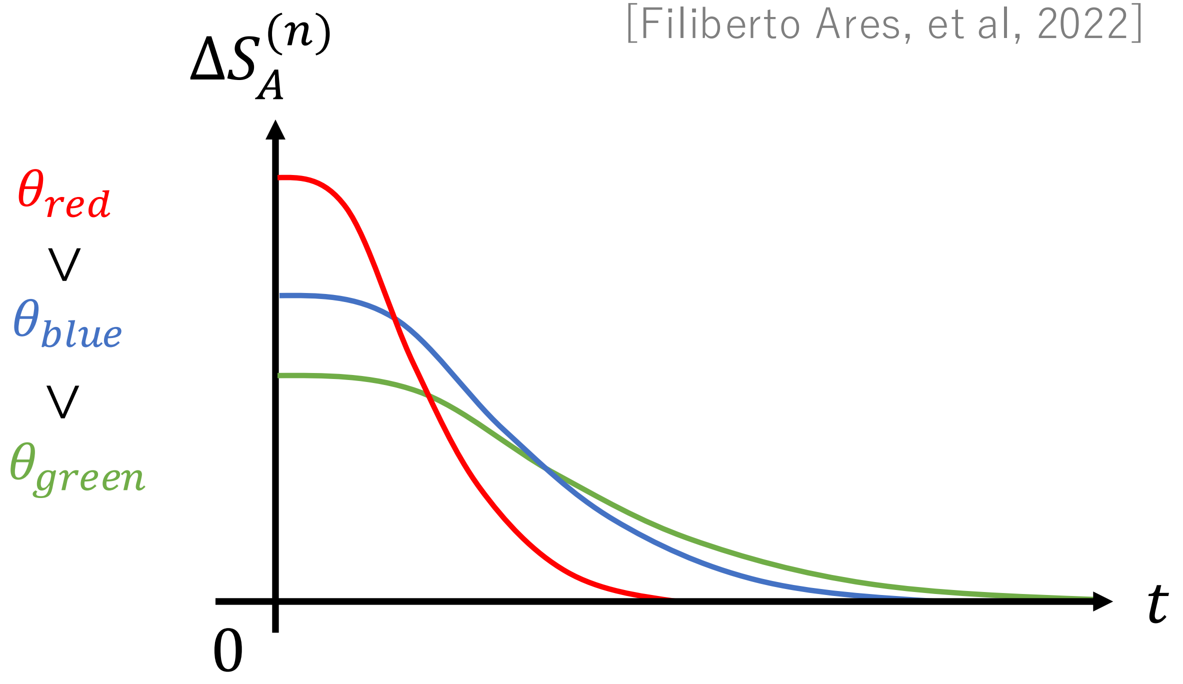
[Filiberto Ares, et al, 2022]



Quantum Mpemba effect :

More the symmetry is initially broken, it is restored faster.

Counterintuitive phenomena which is fined by EA.

Outline

3. Application to QFT and non-invertible symmetry(Ongoing work)

EA for QFT

G : Abelian group of symmetry

$\hat{U}_g$: Symmetry defect with respect to $g \in G$

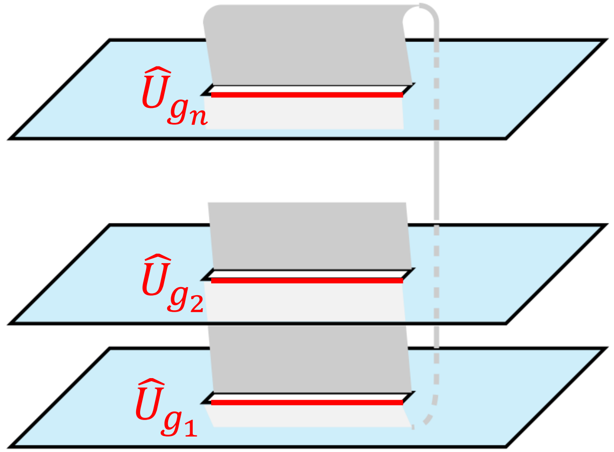
Projector : $\Pi_r = \frac{d_r}{|G|} \sum_{g \in G} \chi_r^*(g) \boxed{\hat{U}_g}$

r : label of irreducible representation of G
d_r: dimension of irrep r
$|G|$: order of group G
$\chi_r(g)$: character of group element $g \in G$

Replica method

$$\left(\rho_{A,Q} \equiv \sum_{q_A} \Pi_{q_A} \rho_A \Pi_{q_A} \right)$$

$$\mathrm{Tr}\left[\rho_{A,Q}^n\right] \sim$$



We can use the same techniques used to calculate entanglement entropy.

3. Application to QFT and non-invertible symmetry(Ongoing work)

<u>Our aim : Generalizing the EA to non-invertible symmetry</u>

Ordinary symmetry
→Group G

$$\hat{U}_g \times \hat{U}_{g'} = \hat{U}_{g''}$$

The inverse $\hat{U}_g^{-1}$ exist

Non-invertible symmetry
→Category C

$$\hat{\mathcal{L}}_a \times \hat{\mathcal{L}}_b = \sum_c N_{a,b}^c \, \hat{\mathcal{L}}_c$$

$\hat{\mathcal{L}}_a$: Symmetry defect of $a \in C$

Lack of inverse $\hat{\mathcal{L}}_a^{-1}$ → non-invertible symmetry

For simplicity, we consider 2d RCFT.

Projector for group

$$\Pi_r = \frac{d_r}{|G|} \sum_{g \in G} \chi_r^*(g) \xrightarrow{\quad \hat{U}_g \quad}$$

Replace
$\hat{U}_g$ to $\hat{\mathcal{L}}_a$ →

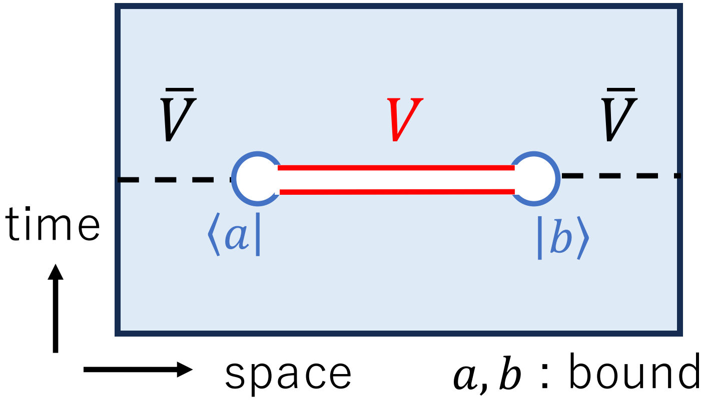
Projector for category

$$\Pi_c = \frac{d_c}{|C|} \sum_{c \in C} \chi_c^*(b) \xrightarrow{\quad \hat{\mathcal{L}}_b \quad}$$

3. Application to QFT and non-invertible symmetry(Ongoing work)

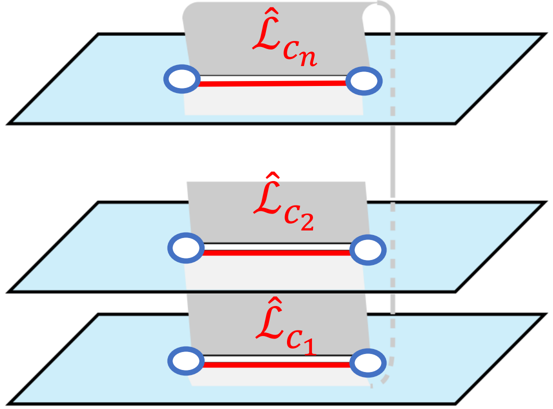
Our setting :
(1+1)d RCFT and set subspace V as follows;

$\rho_V =$



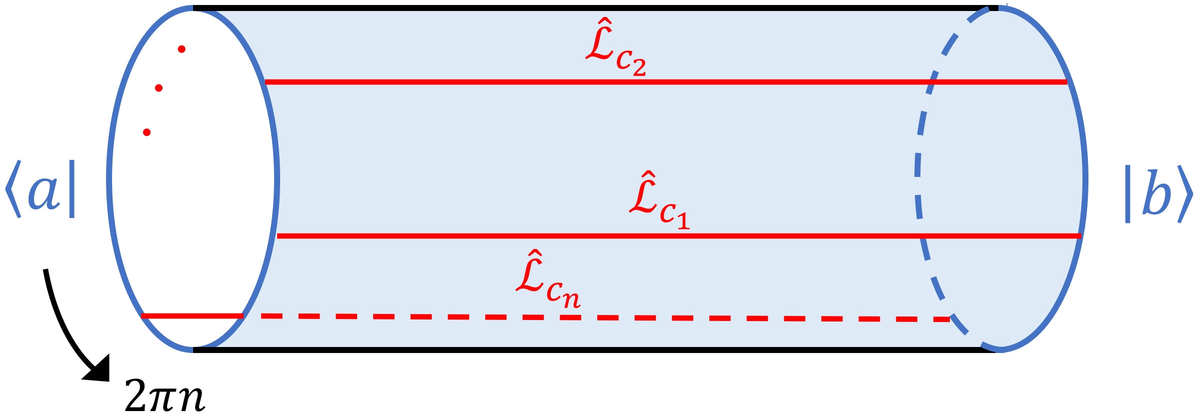
a, b : boundary conditions

$\mathrm{Tr}[\rho_{A,Q}^n] \sim$



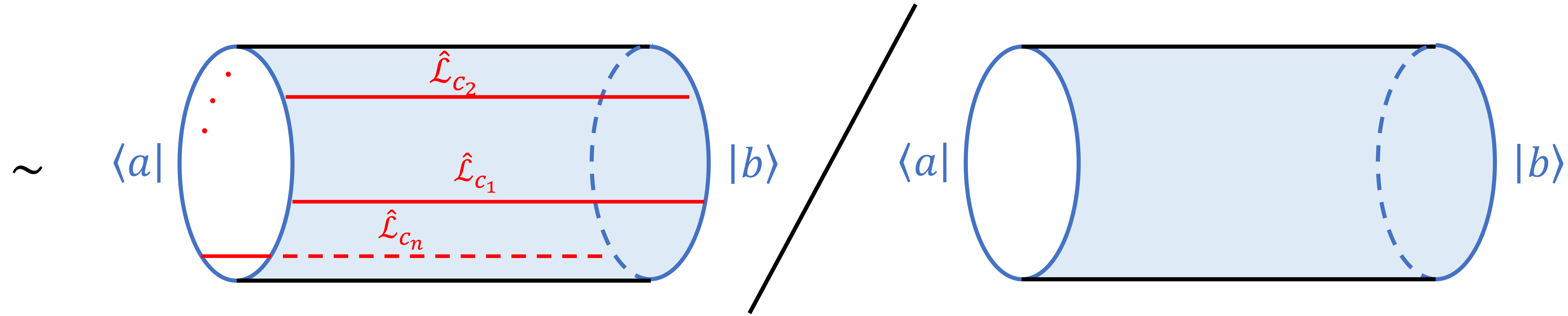
A conformal map reduces this replica manifold to the cylinder.

$\mathrm{Tr}[\rho_{A,Q}^n] \sim$



3. Application to QFT and non-invertible symmetry(Ongoing work)

$$\Delta S_A^{(n)} \sim \mathrm{Tr}[\rho_{A,Q}^n] \,/\, \mathrm{Tr}[\rho_A^n]$$



We are calculating EA using **Tricritical Ising model** (=a 2d RCFT with c=7/10)
=one of the model which shows non-invertible symmetry

The boundary conditions a, b can be symmetric or symmetric → we are investigating⋯

We are exploring whether EA can analyze the breaking of non-invertible symmetry.

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Summary

・Entanglement and symmetry breaking are important concepts in physics.

・ Entanglement quantify how much the symmetry is breaking at the level of subsystem.

(→ Quantum Mpemba effect)

・We are generalizing EA to non-invertible symmetry.

Future direction

・We are actually verifying EA with a concrete model (Tricritical Ising model) which has non-invertible symmetry.

・Currently, we are calculating EA with symmetric or asymmetric boundary conditions.

・We will unveil whether the EA work for non-invertible symmetry as well as for group like symmetry.

Appendix

Appendix

How to detect the continuum symmetry breaking?

$$Q|\Psi\rangle = 0 \ \text{ or } \neq 0$$

Q : Charge of some symmetry

(But $Q|\Psi\rangle$ is ill defined state when $Q|\Psi\rangle \neq 0$)

The mostly used method is to calculate the **order parameter.**

$A(x)$: some operator

$\delta A(x) \equiv i[Q, A(x)]$

$\langle \delta A(x) \rangle = \langle \Psi | \delta A(x) | \Psi \rangle = i \langle \Psi | (Q A(x) - A(x) Q) | \Psi \rangle$

Order parameter as an indicator of symmetry breaking

$\langle \delta A(x) \rangle = 0$ ➡ Symmetric : $Q|\Psi\rangle = 0$

$\langle \delta A(x) \rangle \neq 0$ ➡ Symmetry broken : $Q|\Psi\rangle \neq 0$

Appendix

How to detect the discrete symmetry breaking?

$$U|\Psi\rangle = |\Psi\rangle \text{ or } \neq |\Psi\rangle$$

U : unitary op which implement the transformation of discrete symmetry

Example $\mathbf{Z}_2$: $\phi(x) \mapsto -\phi(x)$

$$-\phi(x) = U^\dagger \phi(x) U$$

$$-\langle\Psi|\phi(x)|\Psi\rangle = \langle\Psi|U^\dagger \phi(x) U|\Psi\rangle = \langle\Psi|\phi(x)|\Psi\rangle \quad \text{If } U|\Psi\rangle = |\Psi\rangle$$

$\langle\Psi|\phi(x)|\Psi\rangle = 0$ (Symmetric)

$\langle\Psi|\phi(x)|\Psi\rangle \neq 0$ (Symmetry broken)