Quantum simulation for screening and confinement in the Schwinger model with topological term*

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*This is Review of the paper[1]

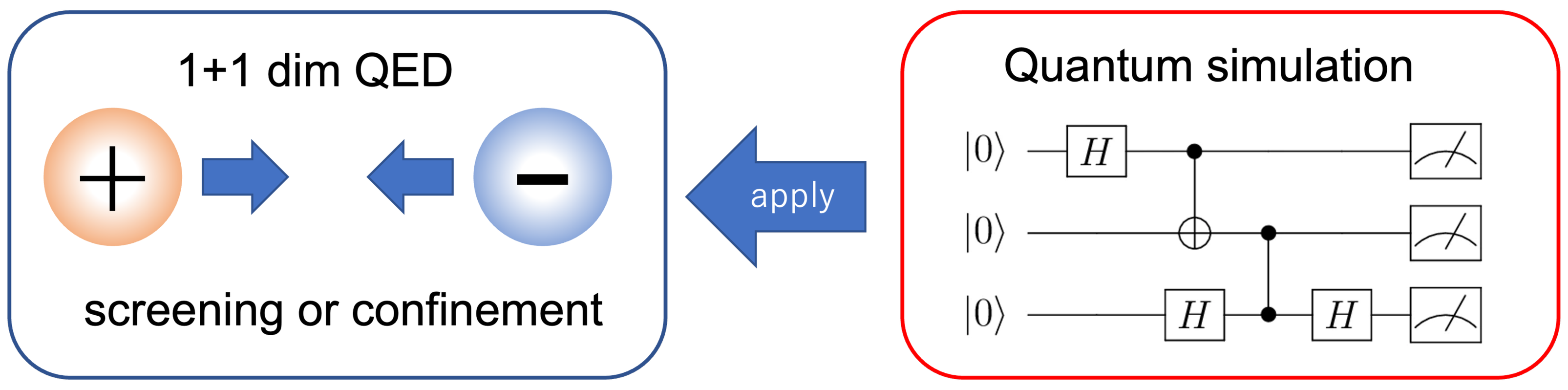
Introduction & Abstract

- The conventional lattice QCD is based on Monte Carlo method and computed by Classical computer.
- But classical computation is difficult in some case because of the sign problem.

New method : Quantum Simulation

- Quantum computers are developing in recent years.
  →It is important to develop methods to analyze QFT by using quantum computer

[Target model] : Schwinger model(1+1dim QED) with theta term



[Results] : Quantum simulation reproduces screening and confinement.
(consistent with analytic results)

What is the Schwinger model ?

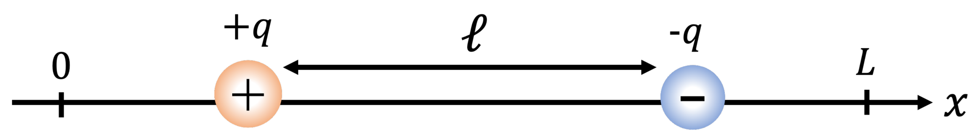
1+1 d QED Lagrangian(continuum)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_\mu + igA_\mu)\psi - m\bar{\psi}\psi$$

$\psi = (\psi_u(x), \psi_d(x))^T$: *Dirac fermion* $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, A_μ: *U*(1) *gauge field*

☆This model shows screening and confinement depending on m and q.

Set probe charge ±q



<Analytical results>

- m=0 : screening (exactly solvable by Bosonization)
- m≠0 : $q \in integer$ → screening ; $q \notin integer$ → confinement (by mass perturbation)

Quantum Simulation Method

Introduce probe charge $\pm q$ by making θ position dependent[2]

$$\theta(x) = \begin{cases} 2\pi q + \theta_0 & (\ell_0 \le x \le \ell_0 + \ell) \\ \theta_0 & (otherwise) \end{cases}$$

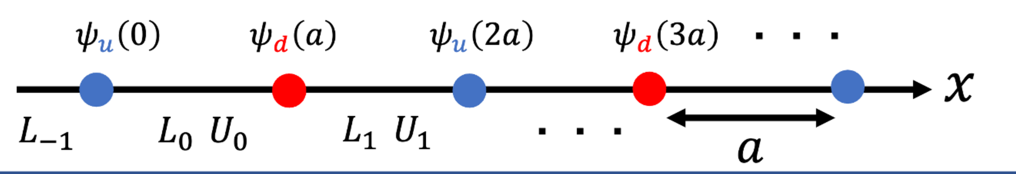
Continuum Hamiltonian ($A_0 = 0$ *gauge*)

$$H = \int dx \left[\frac{1}{2}\left(\Pi - \frac{g\theta}{2\pi}\right)^2 - i\bar{\psi}\gamma^1(\partial_1 + igA_1)\psi + m\bar{\psi}\psi\right]$$

Latticize

Kogut – Susskind formalism

$$\frac{\chi_n}{\sqrt{a}} \leftrightarrow \begin{cases} \psi_u(na) & (n: even) \\ \psi_d(na) & (n: odd) \end{cases} \qquad U_n \leftrightarrow \exp(-iagA^1(na)) \quad L_n \leftrightarrow -\Pi(na)/g$$



Lattice Hamiltonian

$$H = \sum_n \left[\frac{g^2 a}{2}\left(L_n + \frac{\theta_n}{2\pi}\right)^2 - i\frac{1}{2a}(\chi_n^\dagger U_n \chi_{n+1} - \chi_{n+1}^\dagger U_n^\dagger \chi_n) + m(-1)^n \chi_n^\dagger \chi_n\right]$$

There are no gauge d.o.f

- Gauss law constraint and rephase χ_n
- Jordan Wigner transformation

Jordan – Wigner transformation

$$\chi_n = \frac{X_n - iY_n}{2}\prod_{i=0}^{n-1}(-iZ_i) \qquad , X_n, Y_n, Z_n : \textit{Pauli matrices}$$

Qubit Hamiltonian

$$H = J\sum_n\left[\sum_{i=0}^{n}\frac{Z_i + (-1)^i}{2} + \frac{\theta_n}{2\pi}\right]^2 + \frac{w}{2}\sum_n[X_n X_{n+1} + Y_n Y_{n+1}] + \frac{m}{2}\sum_n(-1)^n Z_n$$

Ground state energy $E(\theta_0, q, \ell) \coloneqq \langle GS|H(\theta_0, q, \ell)|GS\rangle$

[Target] : Potential $V(\theta_0, q, \ell) \coloneqq E(\theta_0, q, \ell) - E(0,0,0)$

< Classification >

$$\begin{cases} V \propto \ell \rightarrow \text{confinement} \\ V \sim constant \rightarrow \text{screening} \end{cases}$$

Let's calculate potential by using quantum simulation

Prepare ground state by the adiabatic state preparation

$$H_0 \coloneqq H\big|_{w=0,\theta_0=0,q=0,m=m_0} \Rightarrow |GS_0\rangle = |1010\ldots\rangle, Z|0\rangle = +|0\rangle, Z|1\rangle = -|1\rangle$$

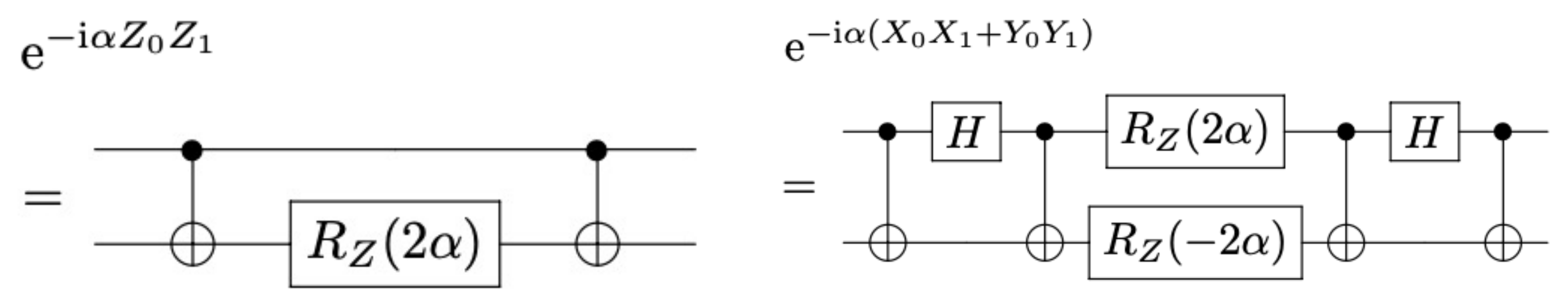
$$|GS\rangle = \lim_{T\to\infty} \mathcal{T}\exp\left(-i\int_0^T H_A(t)\right)|GS_0\rangle \quad , H_A(0) = H_0, H_A(T) = H$$

Suzuki-Trotter decomposition

$$|GS\rangle \approx U(T)U(T-\delta t)\ldots U(\delta t)|GS_0\rangle \quad where\ U(t) = \exp(-iH_A(t)\delta t)$$

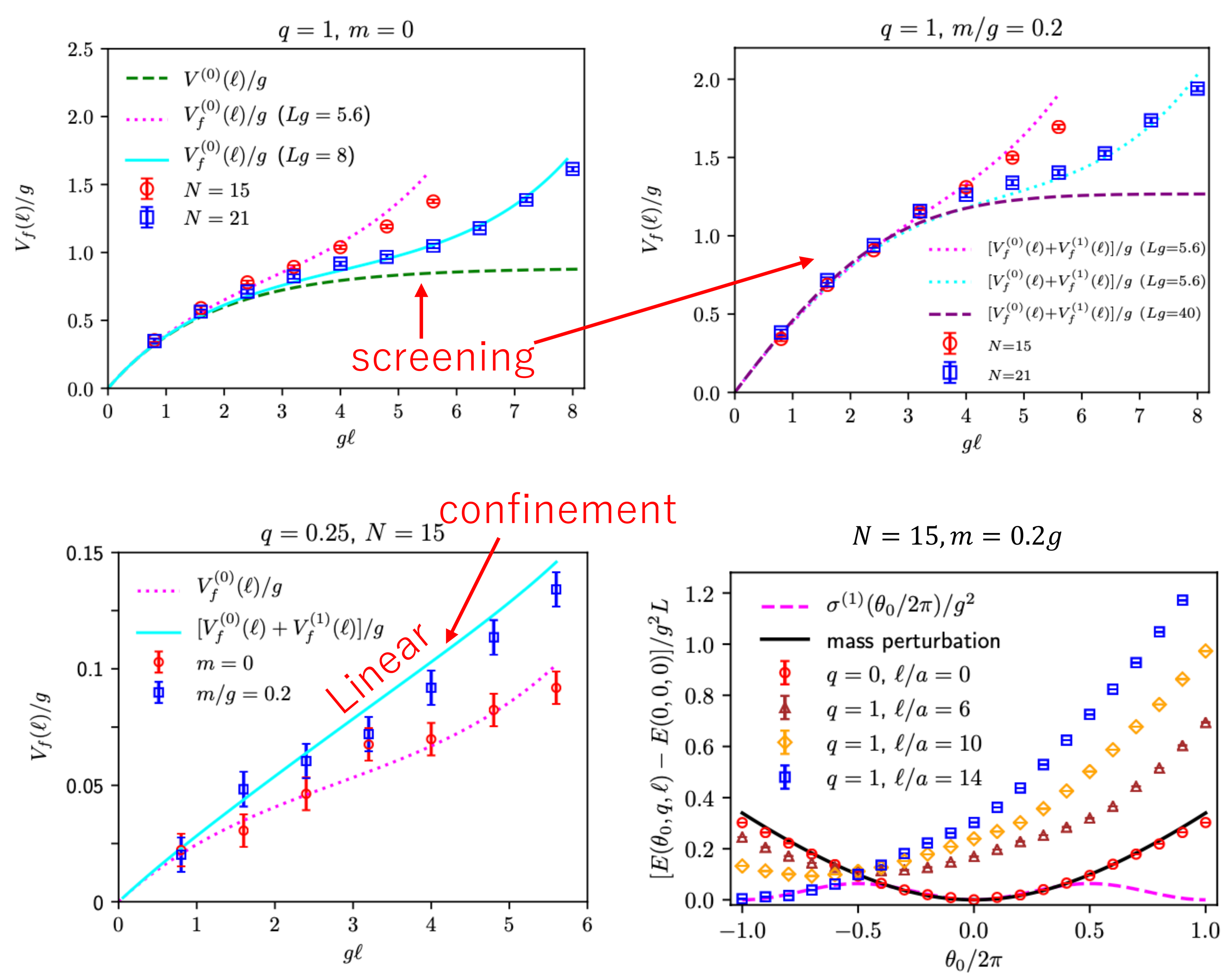
$$\exp(-i(H_1 + H_2)\delta t) = \exp(-iH_1\delta t)\exp(-iH_2\delta t) + \mathcal{O}(\delta t^2)$$

$e^{-i\alpha Z_0 Z_1}$ = ; $e^{-i\alpha(X_0X_1 + Y_0Y_1)}$ =



Simulation results

$a = 0.4g^{-1}, \delta t = 0.3g^{-1}, T = 99g^{-1}, m_0 = 0.5g, , L = (N-1)a, n_{shot} = 10^5$



Summary

- Quantum simulation is a promising alternative to conventional method.
- We apply quantum simulation to Schwinger model and saw screening and confinement by using a simulator on a classical computer. The results were consistent with theoretical analysis.
- We checked the usefulness of quantum simulation.

Reference

[1] Masazumi Honda, Etsuko Itou, Yuta Kikuchi, Lento Nagano, and Takuya Okuda. Phys. Rev. D **105**, 014504, Jan 2022
[2] Sidney Coleman, R.Jackiw, Leonald Susskind. 1975 *Annals Phys.* 93 (1975) 267
[3] Sidney Coleman. 1976, *Annals Phys.* 101, 239-267(1976)
[4] John Preskill 2018 , arXiv:1801.00862 [quant-ph]

Localization 2023, February 13th, at Nagoya U